AN n-DIMENSIONAL EXTENSION OF PICARD'S THEOREM

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We will prove the following theorem:

For each positive integer n, let

$$\rho(n) = \left(\frac{n}{2} + 1\right)^2 + 1 \qquad \textit{if } n \textit{ is even,}$$

$$= \left(\frac{n+1}{2}\right)\left(\frac{n+3}{2}\right) + 1 \quad \textit{if } n \textit{ is odd.}$$

Let C^m be the m-dimensional complex euclidean space and P_nC the n-dimensional complex projective space. Then every holomorphic mapping $x:C^m \to P_nC$ such that $x(C^m)$ omits $\rho(n)$ hyperplanes in general position must reduce to a constant.

Note that $\rho(1)=3$ and P_1C is just the Riemann sphere. So when $m=n=1$, this is exactly the classical theorem of Emile Picard. The first few values of ρ are: $\rho(2)=5, \rho(3)=7, \rho(4)=10, \rho(5)=13, \rho(6)=17, \rho(7)=21, \rho(8)=26, \rho(9)=31, \rho(10)=37$.

Recently, Kobayashi introduced the notion of a hyperbolic (complex) manifold, [3], [4] (cf. also Definition 1.3 of [6]). A hyperbolic manifold has most of the useful analytic properties of a bounded domain in C^n, including the fact that every holomorphic mapping of C into it reduces to a constant. There is the natural question of obtaining hyperbolic manifolds by removing suitable subsets of P_nC. P. A. Griffiths suggested that P_nC minus a *singular* hypersurface of a very high degree might be hyperbolic. On the other hand, P. J. Kiernan has proved in [2] that P_nC minus $2n$ hyperplanes in general position is never hyperbolic. The above result suggests the

CONJECTURE. P_nC minus $\rho(n)$ hyperplanes in general position is hyperbolic.

The determination of the smallest value of $\rho(n)$ for which the above theorem and conjecture remain valid is probably a difficult problem.

The Picard theorem may be generalized in yet another way. We formulate a second

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