MONOTONE OPERATORS AND NONLINEAR INTEGRAL EQUATIONS OF HAMMERSTEIN TYPE

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Communicated by C. B. Morrey, Jr., May 19, 1969

A nonlinear integral equation of Hammerstein type is one of the form

$$u(x) + \int_G K(x, y)f(y, u(y))dy = 0, \tag{1}$$

where G is a measure space with σ-finite measure dy and the unknown function $u(x)$ is defined on G. In operator-theoretic terms, the problem of determining the solutions of the equation (1) with u lying in a given Banach space Y of functions on G can be put in the form of the nonlinear functional equation

$$u + AN(u) = 0 \tag{2}$$

with the linear and nonlinear mappings A and N given by

$$Av(x) = \int_G K(x, y)v(y)dy, \qquad Nu(x) = f(x, u(x)). \tag{3}$$

In the present note, we establish general results on the existence and uniqueness of solutions of equation (2) for the Banach space $Y = X^*$ under appropriate assumptions of weak monotonicity type upon the mappings A and N. We note that Hammerstein equations have an extensive literature which includes Hammerstein [11], Iglisch [12], Golomb [10], Dolph [7], Rothe [18], Vainberg [19], [20], and Krasnosel'skiĭ [16]. The first application of the concept of monotone operator in this problem was made implicitly by Golomb [10] and explicitly by Vainberg [19]. More recent papers applying monotonicity concepts to Hammerstein equations include Dolph-Minty [8], Kolodner [13], Brézis [3], Kolomy [14], [15], Amann [1], [2], de Figueiredo-Gupta [9] and Vainberg [20].

We employ the following definitions: If X is a real Banach space, X^* its conjugate space, we let (w, u) denote the duality pairing between the element w of X^* and the element u of X. A mapping A of X into X^* is said to be monotone if for all u, v in X we have

$$(A(u) - A(v), u - v) \geqq 0.$$

A mapping N of X^* into X is said to be hemicontinuous if it is continuous from each line segment of X^* to the weak topology of X.