ON A NEW FAMILY OF SYMMETRY CODES AND RELATED NEW FIVE-DESIGNS

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For every prime $p \equiv -1$ (3) we define a self-orthogonal $(2p+2, p+1)$ code over GF (3). It can be shown that the group leaving a $(2p+2, p+1)$ code invariant is $\mathrm{PSL}_2(p)$. The minimum weights of the first five codes in the family are determined and lead to new 5-designs.

Let t, r, and n be integers with $t \leqq r \leqq n$. A λ; $t-r-n$ design D is a collection of subsets of the n integers, each subset containing r elements, such that any t-subset of the n integers is contained in the same number λ of subsets in D. Some designs, a 1; $5-6-12$, a 1; $5-8-24$, and a 48; $5-12-24$ associated with the Mathieu groups M_{12} and M_{24}, have been known for a long time. Recently, [1] and [5], 2; $5-6-12$ and 2; $5-8-24$ designs have been found. Using coding theory [2] other 5-designs were found for $n=24$ and $n=48$. We have found new 5-designs for $n=36$ and $n=60$ and a number of r's. Also we found new 5-designs for $n=24$ and $n=48$ which are not equivalent to the ones mentioned above. Two t-designs are called equivalent if there is a permutation of the n integers so that the subsets of D go onto subsets in D.

Let V_{2p+2} be a vector space over GF(3) with a fixed, orthonormal basis. We call a subspace of this space an error correcting code. We define a family of codes of $\dim(p+1)$ (referred to as $(2p+2, p+1)$ codes) by a basis (I, S_p) where S_p is given below.

$$S_p = \begin{array}{c|ccccc|}
 & \infty & 0 & 1 \cdots & j \cdots & (p-1) \\
\infty & 0 & 1 & 1 & 1 & 1 \\
0 & \chi(-1) & \chi(0) & \chi(1) & \chi(j) & \chi(p-1) \\
1 & \chi(-1) & & & & \\
\vdots & & & & & \\
i & \chi(-1) & & & \chi(j-i) & \\
\vdots & & & & & \\
(p-1) & \chi(-1) & & & &
\end{array}$$

where $\chi(0)=0$, $\chi(\text{a square})=1$, $\chi(\text{a nonsquare})=-1$. We refer to the code generated by (I, S_p) as $C(p)$.