A NOTE ON THE NUMBER OF INTEGRAL IDEALS OF BOUNDED NORM IN A QUADRATIC NUMBER FIELD

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Let K be an algebraic number field of degree 2 and $F(n)$ the number of nonzero integral ideals of norm n in K. Define $P(x)$ by

$$\sum_{n \leq x} F(n) = \lambda h x + P(x),$$

where h denotes the class number of K and

$$\lambda = 2^{r_1+r_2}\pi^{r_2}R/(w\sqrt{|\Delta|}),$$

where r_1 is the number of real conjugates, $2r_2$ the number of imaginary conjugates, R the regulator, w the number of roots of unity, and Δ the discriminant of K. It is known that [8, Satz 210] $P(x) = O(x^{1/3})$. On the other hand, Landau [9] also showed that

$$P(x) = \Omega_{\pm}(x^{1/4}).$$

Improvements were made by Szegö and Walfisz [10] and Chandrasekharan and Narasimhan [2], [3]. The former authors showed that if K is imaginary,

$$P(x) = \Omega_{-}(\{x \log x\}^{1/4}),$$

and if K is real

$$P(x) = \Omega_{+}(\{x \log x\}^{1/4}).$$

The latter showed that

$$\limsup_{x\to\infty} \inf P(x)/x^{1/4} = \pm\, \infty. \tag{1}$$

In 1961 Gangadharan [5], improving a method of Ingham, made improvements on (1) for the corresponding problems on $r(n)$, the number of representations of n as the sum of two squares, and $d(n)$, the number of divisors of n. Using Gangadharan's method, we can obtain improvements on (1) for our problem. Before stating this result we must make some definitions.

DEFINITION 1. Let $S_x (x \geqq 2)$ be the set of all real numbers η expressible in the form