ASYMPTOTICS AND RANDOM MATRICES WITH ROW-SUM AND COLUMN SUM-RESTRICTIONS[1]

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1. Introduction. We desire to use the inclusion-exclusion formula for determining asymptotic approximations. This object was first achieved for a special problem by P. Erdös and I. Kaplansky [1]. The following indicates the form of the asymptotic estimate we may obtain.

Let S be an arbitrary finite set, each element of which may be said to possess "properties" from an L-set P of properties $\{P_1, P_2, \cdots, P_L\}$. Let

$$N(P_{i_1}, P_{i_2}, \cdots, P_{i_j})$$

be the number of elements in the set S which possess all the properties of the set $\{P_{i_1}, P_{i_2}, \cdots, P_{i_j}\}$, and possibly more. To define s_j, let

$$N(0)s_j(j!)^{-1} = \sum N(P_{i_1}, P_{i_2}, \cdots, P_{i_j}),$$

where the sum runs over all j-subsets of P. Let us formally define $(s_1)^j = s_j$, with $s_j = 0$ for $j > L$. Then if $E(0)$ denotes the number of elements of S with none of the properties of P, we may formally represent $E(0)$ by

$$E(0) = N(0)e^{-s_1}.$$

This representation is merely the sieve formula or the simple inclusion-exclusion formula in a formal guise. It turns out that for a great many problems of interest, this formal equation is a valid asymptotic approximation when certain restrictions are placed on the properties of P.

Some notation is required for the results which follow. Let $M^n(R, S)$ be the class of $n \times n$ $(0-1)$-matrices with row-sum vector R and column-sum vector S. We denote by r_i or s_i the ith component of the n-length vector R or S respectively. It is always assumed that $\sum r_i = \sum s_i$. We further restrict the vectors R and S so that the number N of integers i in $\{1, 2, \cdots, n\}$ such that $r_i = 0$ or $s_i = 0$ is very small for large n: $N = O(\log n)$. The symbol $M^n(k, k)$ designates the class

[1] This work is a major portion of the author's dissertation for a PhD, received at the Rockefeller University.