RELATIVE HAUPTVERMUTUNG FOR NEIGHBORHOODS OF 1-FLAT SUBMANIFOLDS WITH CODIMENSION TWO

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1. Recently Kirby and Siebenmann have given general solutions of Hauptvermutung [5] and relative Hauptvermutung for neighborhoods of locally flat submanifolds [6]. In this note we announce some results about relative Hauptvermutung for neighborhoods of 1-flat submanifolds with codimension two (compare [11] and [3]).

We shall say that manifold pairs (Q, M) and (Q', M') are *topologically micro-equivalent*, if there are open neighborhoods U, U' of M, M' in Q, Q' and a homeomorphism $h: (U, M) \to (U', M')$, called a *topological micro-equivalence* between (Q, M) and (Q', M'). We shall say that PL manifold pairs (Q, M) and (Q', M') are PL *micro-equivalent*, if there are open neighborhoods V, V' of M, M' in Q, Q' and a PL homeomorphism $g: (V, M) \to (V', M')$, called a PL *micro-equivalence* between (Q, M) and (Q', M').

We shall prove the following

THEOREM A. *Let (Q, M) and (Q', M') be proper PL orientable $(4, 2)$-manifold pairs. Suppose that M is compact and that there is a topological micro-equivalence $h: (U, M) \to (U', M')$. Then there is a PL micro-equivalence $g: (V, M) \to (V', M')$. Further, if $h|M$ is already PL, then we can take g so that $g|M = h|M$.*

In order to extend this result to the higher dimensional case, we need some niceness condition for singularities.

Let (Q, M) be a proper PL $(m+2, m)$-manifold pair. We shall say that M is *locally flat at a point* x of M, if the link pair of x in (Q, M) is PL equivalent to the standard sphere or ball pair. If M is not locally flat at x, then x is called a *singular point* of M, and the link pair is called the *singularity*. We shall say that M is 1-*flat* in Q, if the set of singular points consists of isolated points. Note that if M is 1-flat in Q, the link pair of any point of M in (Q, M) is a locally flat PL $(m+1, m-1)$-sphere or ball pair; i.e. PL$(m-1)$-knot or disk knot, and that if M is of dimension two, then M is always 1-flat in Q. We

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