A GALOIS THEORY FOR A CLASS OF PURELY INSEPARABLE EXPONENT TWO FIELD EXTENSIONS

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Introduction. A Galois theory for purely inseparable exponent one field extensions was developed by N. Jacobson [2] in 1944. He accomplished this by characterizing the finite dimensional subalgebras $\mathrm{Der}_k(K)$ of $\mathrm{Der}(K)$, where $\mathrm{Der}(K)$ is the Lie algebra of derivations on K, k is a subfield of K, and $\mathrm{Der}_k(K)$ is the subalgebra of $\mathrm{Der}(K)$ consisting of those derivations that are zero on k. It was conjectured that higher derivations might provide an extension of the theory to field extensions of higher exponent. The purpose of this note is to describe such an extension. The author believes that the exponent two case is of sufficient interest to justify its presentation before the general exponent N case is developed. The problem of extending the theory to exponent N appears to be nontrivial; the author's efforts in this area thus far have been unsuccessful.

Let K be a field of characteristic $p \neq 0$ or 2 and let $H^p(K)$ denote the set of all higher derivations of K having length p; that is, sequences of additive mappings (d_i) of K into itself such that for all x and y in K and $n=0, 1, \cdots, p$: $d_n(xy) = \sum \{d_i(x)d_j(y) \mid i+j=n\}$ and d_0 is the identity mapping on K. $H_p(K)$ is a group under $(d_i)(e_i) = (f_i)$ where $f_i = \sum\{d_j e_n \mid j+n=i\}$. $H_p(K)$ is also closed under a type of scalar multiplication by elements of K; this is defined by $a(d_i) = (a^i d_i)$ where $a^i d_i = (a^i)_L d_i$ and $a \in K$. If k is a subfield of K, $H^p_k(K)$ will denote the subset of those (d_i) in $H^p(K)$ with the property that d_i restricted to k is zero for $i=1, 2, \cdots, p$, $H^p_k(K)$ is a subgroup of $H^p(K)$ and is closed under scalar multiplication by elements of K. In the higher derivation setting, Jacobson's result was the characterization of the finitely K-generated subgroups $H^1_k(K)$ of $H^1(K)$.

In this note we give an intrinsic characterization of those subgroups $H^p_k(K)$ of $H^p_k(K)$ having the property that they are finitely K-generated; that is, there is a finite subset S of $H^p_k(K)$ such that the minimal subgroup of $H^p(K)$ containing S and closed under scalar multiplication is $H^p_k(K)$. The result can then be used to provide a Galois type correspondence between these subgroups of $H^p(K)$ and subfields k of K satisfying: $[K:k] < \infty$, exponent of $K/k = 2$, and K is the tensor product of simple extensions of k. Only sketches of proofs are given.