# OCTONION PLANES IN CHARACTERISTIC TWO

BY JOHN R. FAULKNER[1]

Communicated by Nathan Jacobson, March 7, 1969

1. Roughly speaking an octonion plane $\mathcal{P}(\mathfrak{O})$ is a plane coordinatized by an octonion (Cayley-Dickson) algebra $\mathfrak{O}$. The most successful approach to these planes has been made by first constructing a reduced exceptional simple Jordan algebra $\mathfrak{J}=\mathfrak{H}(\mathfrak{O}_3, \gamma)$ and then using $\mathfrak{J}$ to define $\mathcal{P}(\mathfrak{O})$. The results on $\mathcal{P}(\mathfrak{O})$ in [2], [7], [8], [10] and [13] for $\mathfrak{O}$ an octonion division algebra and those in [9], [11] and [12] for split $\mathfrak{O}$ were obtained along these lines. However, in all of these papers the characteristic of the field is not two. In the present paper, we give a definition of octonion planes based on quadratic Jordan algebras. This definition is valid for all characteristics and both types of octonion algebras. We also indicate how most of the results mentioned above can be derived in this general setting.

We refer the reader to McCrimmon [4] for a definition of a quadratic Jordan algebra. Recall also that the set $\mathfrak{H}(\mathfrak{O}_3, \gamma)$ of 3 by 3 matrices with entries in $\mathfrak{O}$ which are symmetric with respect to the involution $x\mapsto\gamma^{-1}\bar{x}^t\gamma$ (where $\gamma=\operatorname{diag}\{\gamma_1, \gamma_2, \gamma_3\}$, $0\neq\gamma_i\in\Phi$, the field) can be endowed with the structure of a quadratic Jordan algebra. (See [5].) We shall use the notation of [4] and [5] with the exception that we write our operators on the right and define $zV_{x,y}=\{zxy\}$ $=xU_{z,y}$. We remark that $U_{x,y}=U_{x+y}-U_x-U_y$ where in characteristic not two $U_x=2R_x^2-R_{x^2}$ where $yR_x=\frac{1}{2}(xy+yx)$. Also $x^\#$ in [5] is roughly the adjoint of the matrix $x$ and $x\times y=(x+y)^\#-x^\#-y^\#$.

2. Let $\mathfrak{J}=\mathfrak{H}(\mathfrak{O}_3, \gamma)$ and let $x_*$ and $x^*$ be two copies of $\{\alpha x \mid 0\neq\alpha\in\Phi\}$ where $x\in\mathfrak{J}$ is of rank one; i.e., $x\neq 0$ but $x^\#=0$. We define the octonion plane $\mathcal{P}(\mathfrak{J})$ to have points $x_*$ and lines $y^*$, where $x$ and $y$ are of rank one, and relations (cf. [9])

(1) $x_*\mid y^*$, $x_*$ *incident* to $y^*$, if $V_{y,x}=0$,
(2) $x_*\simeq y^*$, $x_*$ *connected* to $y^*$, if $T(y, x)=0$,
(3) $x_*\simeq y_*$, $x_*$ *connected* to $y_*$, if $y\times x=0$,
(4) $x^*\simeq y^*$, $x^*$ *connected* to $y^*$, if $y\times x=0$.

[1] These results are contained in the author's doctoral dissertation written under the guidance of Professor N. Jacobson at Yale University. A more detailed paper is forthcoming. The author was a National Science Foundation Graduate Fellow while at Yale.