FUNCTION THEORETIC METHODS IN THE THEORY OF BOUNDARY VALUE PROBLEMS FOR GENERALIZED METAHARMONIC FUNCTIONS[1]

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1. **Introduction.** Although existence and uniqueness theorems for linear elliptic partial differential equations in a domain D with coefficients continuous in D have been known for some time, similar results for equations whose coefficients have singularities in the domain under consideration are practically unknown. Recently attention has been given to a class of singular equations which appear frequently in both pure and applied mathematics and are known as generalized axially symmetric partial differential equations [3], [5], [8]. Just as a thorough knowledge of the Laplace and Helmholtz equation guided the attack on linear elliptic equations with continuous coefficients, it is hoped that a better understanding of generalized axially symmetric equations will give insight towards developing a theory of elliptic equations with singular coefficients.

We wish to announce in this note a uniqueness theorem for the exterior Dirichlet problem for the generalized axially symmetric metaharmonic equation

$$(1) \qquad L_{\lambda,s}[u] \equiv \frac{\partial^2 u}{\partial x_1^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} + \frac{\partial^2 u}{\partial \rho^2} + \frac{s}{\rho}\frac{\partial u}{\partial \rho} + \lambda^2 u = 0$$

where $s > -1$, $s \neq 0$, and $\lambda > 0$ [5]. This is the first time a uniqueness theorem has been obtained for a singular elliptic partial differential equation in more than two variables whose coefficients are singular in its domain of definition, such work in the present case having been delayed due to an insufficient knowledge of certain areas of the theory of several complex variables. Our result depends on first using the Hadamard-Gilbert Theorem [5] to solve the classical expansion problem for Appell series and to then apply this along with Vitali's theorem for several complex variables [6] to obtain the desired uniqueness theorem.

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