TAMING CODIMENSION THREE EMBEDDINGS[1]

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1. Taming embeddings of certain polyhedra. If $P \subset K$ are polyhedra, we say there is an *elementary $C(X)$-construction* from P to K, written $P^{C_e(X)} \nearrow K$, if there exists a pair $(\mathcal{C}(\chi), \chi)$ which is PL homeomorphic to $(C(X), X)$, where $C(X)$ denotes the cone over a polyhedron X, such that

(1) $K = P \cup \mathcal{C}(\chi)$,

(2) $\chi = P \cap \mathcal{C}(\chi)$, and

(3) P is link collapsible on χ and $\mathrm{Cl}(P - \chi) = P$. A polyhedron K is said to be *$C(B)$-constructible* from P, written $P^{C(B)} \nearrow K$, if there is a finite sequence, $P^{C_e(X_1)} \nearrow P_1{}^{C_e(X_2)} \nearrow P_2{}^{C_e(X_3)} \nearrow \cdots {}^{C_e(X_p)} \nearrow K$, of elementary constructions from P to K where each X_i is a PL ball. Similarly, K is said to be *$C(S)$-constructible* from P, written $P^{C(S)} \nearrow K$, if there is such a finite sequence where each X_i is a PL sphere. A *P-nice polyhedron* K has the property that $P^{C(B)} \nearrow L^{C(S)} \nearrow K$. A B-nice polyhydron, B a ball, is simply called a *nice polyhedron.* An embedding f of a polyhedron P into a polyhedron Q is *isotopically tame* if there is an isotopy e_t of Q such that $e_0 = 1$ and $e_1 f$ is PL. The embedding f is *ϵ-tame* if for any $\epsilon > 0$ there is an isotopy e_t of Q such that $e_0 = 1$, $e_1 f$ is PL, dist $(x, e_t(x)) < \epsilon$ for all $x \in Q$ and $e_t | Q - N_\epsilon(f(P)) = 1$.

THEOREM 1. *Let K be a P-nice polyhedron such that* $\dim(\mathrm{Cl}(K-P))$ $\leqq n-3$, $n \geqq 5$, *and let $f: K \to \mathring{Q}^n$ be an embedding of K into the interior of the PL n-manifold Q which is locally flat on the open simplexes of some triangulation and is such that $f|P$ is isotopically tame. Then, f is isotopically tame and if $f|P$ is PL, the taming isotopy is fixed on $f(P)$.*

THEOREM 2. *If $f: I^k \to \mathring{M}^n$, $n \geqq 5$, (no restriction on k) is a locally flat embedding of I^k into the interior of the PL n-manifold M, then f is ϵ-tame.*

THEOREM 3. *If $f: S^k \to \mathring{M}^n$, $n \geqq 5$, $n-k \geqq 3$, is a locally flat embedding of S^k into the interior of the PL n-manifold M, then f is ϵ-tame.*

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[1] This research overlaps the author's doctoral dissertation which was written under the direction of Professor J. C. Cantrell at the University of Georgia. Some of the more recent results of this announcement (in particular the theorems concerning the taming of cells and spheres) are joint results with Professor Cantrell.

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