SOME LINEAR TOPOLOGICAL PROPERTIES OF L^∞ OF A FINITE MEASURE SPACE

BY HASKELL P. ROSENTHAL[1]

Communicated by Bertrom Yood, February 19, 1969

We are interested here in isomorphic invariants of the various Banach spaces associated with the spaces $L^\infty(\mu)$ for finite measures μ. (Throughout, "μ" and "ν" denote arbitrary finite measures on possibly different unspecified measureable spaces.) We classify the spaces $L^\infty(\mu)$ themselves up to isomorphism (linear homeomorphism) in §3, where we also obtain information on the spaces A and A^* for subspaces A of $L^1(\mu)$. In §2, we give a short proof of a result (Corollary 2.2) which simultaneously generalizes the result of Pełczyński that $L^1(\mu)$ is not isomorphic to a conjugate space if μ is nonpurely atomic [7], and the result of Gel'fand that $L^1[0, 1]$ is not isomorphic to a subspace of a separable conjugate space (c.f. [8]). We also obtain there that an injective double conjugate space is either isomorphic to l^∞ or contains an isomorph of $l^\infty(\Gamma)$ for some uncountable set Γ, if it is infinite dimensional. (Henceforth, all Banach spaces considered are taken to be infinite dimensional. Also, we recall that a Banach space is called injective if every isomorphic imbedding of it in an arbitrary Banach space Y is complemented in Y.)

We include brief proofs of some of the results. Full details of the work announced here and other related work will appear in [11].

1. **Preliminary results.** $M(S)$ denotes the space of all regular bounded scalar-valued Borel measures on S. (Throughout, "S" denotes an arbitrary compact Hausdorff space.)

LEMMA. *Let A be a closed subspace of $M(S)$. Then either there exists a positive $\mu\in M(S)$ such that $A\subset L^1(\mu)$ (that is, every member of A is absolutely continuous with respect to μ), or A contains a subspace complemented in $M(S)$ and isomorphic to $l^1(\Gamma)$ for some uncountable set Γ.*

It is easily seen that these possibilities are mutually exclusive. (In fact it follows from known results that for uncountable Γ, $l^1(\Gamma)$ is not isomorphic to a subspace of any WCG Banach space as defined in §2.)

The lemma is proved by using the Radon-Nikodým theorem and a generalization of an argument of Köthe [5]. A consequence of its proof is the

[1] This research was partially supported by NSF-GP-8964.