TRIANGULATION OF MANIFOLDS. I

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I. **Introduction.** The purpose of this note is to outline a proof that almost every compact manifold M of dimension greater or equal to six, (or dimension greater or equal to five if $\partial M = 0$) can be triangulated as a combinatorial manifold. The word "almost" refers to two types of restrictions we impose on M. First that $\pi_1(M)$ be sufficiently nice. Secondly, and more unpleasantly we require $H^4(M; Z_2) = 0$ and $H^3(\partial M; Z_2) = 0$. Since the existence of triangulation implies uniqueness, a consequence of our result is a form of the Hauptvermutung which covers some cases previously not done. For example $\mathrm{Wh}(\pi_1(M)) = 0$.

The proofs of the announced theorems make use of a number of recent results, and thus represent a collective mathematical achievement. The fundamental breakthrough is due to Kirby in [3], who first established what we refer to as Lemma 2, conjectured Proposition 3, and recognized its importance for triangulating manifolds and for the annulus conjecture. Other important geometric ingredients are due to Lees [4] and Lashof [5]. A self-contained and straightforward geometric proof of the theorem based on these results is possible if one is willing to assume M is 4-connected. The more general form stated above depends on delicate and difficult algebraic results of Hsiang and Shaneson [2], Shaneson [7], and Wall [8]. We understand that Kirby and Siebenmann also have obtained similar results.

In this note, I, we outline a proof of our main technical result, Theorem 3. In II, we show how this implies the results of the first paragraph.

The following is a useful trick, first observed by Kervaire.

LEMMA 1. *Let $\phi: X \times R \to Y \times R$ be a homeomorphism where X, Y are compact spaces and R the real numbers. Then there exist a homeomorphism $\psi: X \times S^1 \to Y \times S^1$ such that for $\epsilon > 0$ and sufficiently small the following diagram commutes:*

$$\begin{array}{ccc} X \times [-\epsilon, \epsilon] & \xrightarrow{\phi} & Y \times R \\ \downarrow 1 \times \exp & & \downarrow 1 \times \exp \\ X \times S^1 & \xrightarrow{\psi} & Y \times S^1 \end{array}$$

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