DESUSPENDABILITY OF FREE INVOLUTIONS ON BRIESKORN SPHERES

BY CHARLES H. GIFFEN[1]

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The Brieskorn manifolds K_q^{2m-1} which we consider are the following. Let $V_q(m)$ be the set of points $(z_0, \cdots, z_m) \in C^{m+1}$ such that

$$z_0^q + \sum_{i=1}^{m} z_i^2 = 0,$$

and let $S^{2m+1} \subset C^{m+1}$ be the unit sphere; then

$$K_q^{2m-1} = V_q(m) \cap S^{2m+1}$$

is a smooth (actually, weakly complex) codimension 2 submanifold of S^{2m+1} (c.f. [2]). If we set

$$F_q^{2m} = \left\{(z_0, \cdots, z_m) \in S^{2m+1} \mid z_0^q + \sum_{i=1}^{m} z_i^2 \geqq 0\right\},$$

then (S^{2m+1}, K_q^{2m-1}) is a fibered pair over S^1 in the sense of [4] with parallelizable fiber F_q^{2m}; moreover, K_{2d+1}^{4k+1} is a homotopy sphere, and the Arf invariant $c(F_{2d+1}^{4k+2}) \in Z_2$ is (cf. [8]):

$$(*) \qquad \begin{array}{ll} 0 & \text{if } d \equiv 0, 3 \bmod 4, \\ 1 & \text{if } d \equiv 1, 2 \bmod 4. \end{array}$$

The involution T: $C^{m+1} \to C^{m+1}$ given by $T(z_0, z) = (z_0, -z)$ is fixed point free on K_q^{2m-1}. The weakly complex bordism classification of (T, K_q^{2m-1}) is considered in [5]. We shall have need here of the invariant codimension 1 submanifold K_q^{2m} of K_q^{2m+1} given by

$$K_q^{2m} = \left\{(z_0, \cdots, z_{m+1}) \in K_q^{2m+1} \mid z_{m+1} \text{ imaginary}\right\}.$$

Regarding $C^{m+1} \subset C^{m+2}$ in the obvious way, we have

$$(T, K_q^{2m-1}) \subset (T, K_q^{2m}) \subset (T, K_q^{2m+1}).$$

Setting $Q_q^n = K_q^n/T$, we see that there is a homotopy equivalence of degree ± 1

$$h\colon\ Q_{2d+1}^{4k+1} \to P^{4k+1}$$

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