# ABOUT THE COHOMOLOGY RING OF A FINITE ABELIAN GROUP

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Since the structure of finite abelian groups is so simple, one would expect that the cohomology ring $H(G, R)$ of a finite abelian group $G$ over a commutative ring $R$ to be completely and explicitly known, particularly when the action of $G$ on $R$ is trivial and the ring $R$ is some reasonable ring, say the ring $\boldsymbol{Z}$ of integers. Indeed considerable work has been accomplished in the computation of a homology of finite abelian groups in the context of Eilenberg-MacLane spaces by Eilenberg and MacLane [4]; a remarkable algebraic theory around this topic was built up by Cartan [2]. Further analysis of the homology ring of a finite abelian group is presented in [9]. However, as far as we can see, there still is no explicit and functorial description of $H(G, R)$ in the general case. The following observations still do not fill this gap, but they contribute some new facets and perhaps offer a more direct approach to some results which are in the literature. Our approach is designed specifically to allow for easy generalization to the cohomology ring of a compact abelian group, which we discuss in this journal [6]. The full details and the proofs will appear elsewhere.

Rather than to give too many technical details, we describe the essential features of our approach and try to point out where it differs from other methods.

Each finite abelian group $G$ decomposes into a direct sum of cyclic subgroups $G_1 \oplus \cdots \oplus G_n$ of orders $z_i$, $i=1, \cdots, n$, such that $z_i | z_{i+1}$, $0<i<n$; this standard decomposition is not unique, even though the $z_i$ are. Since we want a description of $H(G, R)$ which is functorial in $G$(and $R$), one is faced with two conflicting objectives: firstly, the final results must not depend on the given product decomposition; secondly, one practically has to use the structure theorem for $G$ to obtain any reasonably explicit description for $H(G, R)$. Thus one attempts to exploit a standard decomposition initially and then to eliminate the dependence of the direct sum decomposition by functorial methods. Thus our first step is to describe explicitly an

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