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A GENERAL MEAN VALUE THEOREM[1]

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We present here in general terms the idea of the mean of a function relative to a "weight function" $w(\xi, \nu)$, special instances and applications appearing elsewhere [1], [2].

1. **The weight function.** If $X=[h, k]$ is a real interval, (I, A, μ) a finite measure space with $\mu(I)=1$, and $w(\xi, \nu)$ a nonnegative function on $X \times I$ which, for each ν of I, is measurable, and positive a.e. on X, then the indefinite integral

$$W(x, \nu) = \int_h^x w(\xi, \nu)\,d\xi \tag{1}$$

is defined on $X \times I$, and the function

$$\mathcal{W}(x) = \int_I W(x, \nu)\,d\mu, \qquad x \in X$$

which we assume to exist, is continuous and strictly increasing on X, as is $W(x, \nu)$ for each ν.

2. **The mean of a function.** Let $x(\nu)$ be any μ-integrable function on I to X for which the integral functional

$$\mathcal{W}_x = \int_I W(x(\nu), \nu)\,d\mu$$

exists. Let x_u be the *essential* upper bound of $x(\nu)$ on I, i.e., the g.l.b. of all real x for which $\mu\{\nu \mid x(\nu) > x\} = 0$, the essential lower bound

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