AN ABELIAN p-GROUP WITHOUT THE ISOMORPHIC REFINEMENT PROPERTY[1]

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It is well known that a countable reduced abelian p-group G has the isomorphic refinement property, i.e. any two direct decompositions of G have isomorphic refinements, if and only if G has no elements of infinite height. In the uncountable case the situation is characteristically unclear. There do exist uncountable reduced p-groups with nonzero elements of infinite height having the isomorphic refinement property; any p-group whose first Ulm factor is torsion-complete and second Ulm factor is cyclic is such a group. And for uncountable p-groups with no elements of infinite height there exist sufficient conditions for the isomorphic refinement property, e.g., those of Crawley [3] and Warfield [4]. Yet the question has remained whether the isomorphic refinement property is possessed by all such groups. Here we answer this question in the negative by showing that *there exists an abelian p-group with no elements of infinite height having two direct decompositions that do not admit isomorphic refinements.*

The foregoing result is actually obtained as a corollary to the following theorem: *there exist three abelian p-groups K, L and M, each with no elements of infinite height, such that no Ulm invariant of K exceeds* 1, $K\oplus L\cong K\oplus M$, *yet* $L\not\cong M$. In particular, this shows that the cancellation theorem of Crawley [2] does not extend to the uncountable case.

To see how our first result follows from the second, let K, L and M be as above, and assume further that K has the isomorphic refinement property. We will show that the direct decompositions $K\oplus L\cong K\oplus M$ do not have isomorphic refinements. If they do, there exist groups K_i, L_i, K_i', M_i $(i=1, 2)$ such that

$$K = K_1 \oplus K_2 = K_1' \oplus K_2', \qquad L = L_1 \oplus L_2, \qquad M = M_1 \oplus M_2,$$

and

$$K_1 \cong K_1', \qquad K_2 \cong M_1, \qquad L_1 \cong K_2', \qquad L_2 \cong M_2.$$

Now by assumption, K has the isomorphic refinement property, and therefore there exist groups K_{ij}, K_{ij}' $(i, j=1, 2)$ such that

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