ON CHARACTERISTIC CLASSES OF FLAT RIEMANNIAN MANIFOLDS

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The "geometric" subject of compact, connected, flat Riemannian manifolds has long ago been "reduced" to purely algebraic questions. Such manifolds are determined (up to a connexion preserving diffeomorphism) by the isomorphism class of their fundamental groups. A precise characterization of those groups which arise in this way is also classical. See [4] or [8] for a convenient modern account. Despite this, apparently simple geometric questions remain unanswered. One such which has intrigued the author and others is this: does every compact flat Riemannian manifold bound a manifold of one higher dimension? If not, which elements of the cobordism ring can be represented by such manifolds? It is known [6], [7], of course, that this question turns on the values of the Stiefel-Whitney numbers and, in the orientable case, the Pontrjagin numbers of the manifold.

A few general facts are easy to establish. Since the structure group of the tangent bundle of any Riemannian manifold is reducible to its holonomy group and since the holonomy group of a compact connected flat Riemannian manifold is *finite*, the Pontrjagin classes are all torsion classes. It follows that the Pontrjagin numbers vanish (orientability, of course, must be assumed for the statement even to make sense). Thus, such a manifold determines a torsion element in the oriented cobordism ring. A similar argument shows that if the order of the holonomy group is *odd*, the Stiefel-Whitney classes vanish. Thus, in this case the manifold automatically bounds, and if it is orientable, bounds an orientable manifold. Such optimism that these simple arguments may arouse is restrained by the example [1] of Auslander and Szczarba showing that the Stiefel-Whitney classes themselves do not always vanish.

In view of the sharp dependence on the holonomy group, it seems reasonable to consider for each finite group, Φ, the class of compact connected flat Riemannian manifolds having holonomy group isomorphic to Φ . . . Φ-manifolds for short.

Recall that the tangent bundle of any differentiable manifold, X, determines the *characteristic subalgebra* of the cohomology algebra, $H^*(X; R)$. Here R is any ring of coefficients; when R is the integers modulo two, the characteristic subalgebra is generated by the Stiefel-Whitney classes. One geometric consequence of the algebraic theorem