k-MERSIONS OF MANIFOLDS[1]

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Let M^n be an n-dimensional C^∞ manifold and W^p be a p-dimensional C^∞ manifold. A C^∞ mapping f: $M^n \to W^p$ is called a k-mersion if its rank is greater than or equal to k everywhere. The set of k-mersions, endowed with the C^1 topology, is denoted $R(M^n, W^p; k)$. A k-regular homotopy between k-mersions f and g is a continuous mapping F: $I \to R(M^n, W^p; k)$ such that $F(0) = f$ and $f(1) = g$.

A k-bundle map, ψ: $TM^n \to TW^p$ between the tangent spaces of M^n and W^p is a continuous fibre preserving mapping such that the restriction of ψ to any fibre is a linear map of rank at least k. The space of k-bundle maps with the compact open topology is denoted $T(M^n, W^p; k)$.

An n-mersion is an immersion, and an n-regular homotopy is usually called a regular homotopy. In 1958 and 1959, Smale [4], [5] published papers classifying immersions of spheres in Euclidean spaces. Smale proved that if $n < p$, the regular homotopy classes of immersions of S^n in E^p are in one to one correspondence with the homotopy classes of sections of S^n into the bundle associated with TS^n whose fibre is the Stiefel manifold $V_{p,n}$ of n frames in p-dimensional Euclidean space. Smale obtained this classification by proving a stronger result, namely, that the map d: $R(S^n, E^p; n) \to T(S^n, E^p; n)$ defined by $d(f) = df$ is a weak homotopy equivalence if $n < p$. His proof was based on the diagram

$$\begin{array}{ccc} R(S^n, E^p; n) & \xrightarrow{d} & T(S^n, E^p; n) \\ \downarrow i^* & & \downarrow j^* \\ R(D^n, E^p; n) & \xrightarrow{d} & T(D^n, E^p; n) \end{array} \tag{1}$$

where D^n is identified with a hemisphere of S^n, and i^* and j^* are restriction maps. The main step in the proof consists of showing that i^*

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