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EXTREMAL PROBLEMS FOR FUNCTIONS OF BOUNDED BOUNDARY ROTATION[1]

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Communicated by Alberto P. Calderón, April 19, 1967

1. **Preliminaries.** Let V_k denote the class of analytic functions in $D=\{z: |z|<1\}$ which have there the representation

$$f(z) = \int_0^z \exp\left(-\int_0^{2\pi} \log(1-\zeta e^{-i\theta})d\psi(\theta)\right)d\zeta \tag{1}$$

where $\psi(\theta)$ is a real valued function of bounded variation for $0\leqq\theta<2\pi$, satisfying there the conditions

$$\int_0^{2\pi} d\psi(\theta) = 2, \qquad \int_0^{2\pi} |d\psi(\theta)| \leqq k.$$

V_k is the class of analytic functions in D which have boundary rotation bounded by $k\pi$. Thus, V_k consists of those functions $f(z) = z+a_2z^2+\cdots$ which are analytic and satisfy $f'(z)\neq 0$ in D, and map D onto a domain having boundary rotation bounded by $k\pi$.

Briefly, the boundary rotation of a schlicht domain G with continuously differentiable boundary curve is the total variation of the direc-

[1] Research partially sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. AF-AFOSR 1077-66.

[2] The author wishes to express his gratitude to Professor Harry E. Rauch for his many helpful conversations during this research.