SYMMETRY IN NONSELFADJOINT STURM-LIOUVILLE SYSTEMS

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Suppose that $a<b$ and C is the inner product space of all continuous real-valued functions on $[a, b]$ such that $\|f\|=(\int_a^b f^2)^{1/2}$ if f is in C. Denote by each of p and q a member of C such that $p(x)>0$ for all x in $[a, b]$. Denote by each of W and Q a real 2×2 matrix and denote by C' the subspace of C consisting of all f in C such that $(pf')'-qf$ is in C and

$$W\begin{bmatrix} f'(a) \\ p(a)f'(a)\end{bmatrix}+Q\begin{bmatrix} f'(b) \\ p(b)f'(b)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

Denote by L the transformation from C' to C such that if f is in C', then $Lf=(pf')'-qf$. Assume for the remainder of this note that L has an inverse T. The purpose of this note is to point out that if $T\neq T^*$ it is nevertheless true that T is very closely related to a symmetric operator. Specifically T is a dilation (*via* the two-dimensional space of solutions to the homogeneous equation) of a symmetric operator. This fact permits an analysis of T using the theory of completely continuous symmetric operators. This suggests a worthwhile alternative to the approach taken in [1, Chapter 12], in which the general theory of completely continuous operators is used.

Denote by S' the subspace of C consisting of all f so that $(pf')'-qf=0$ and denote by S the orthogonal complement in C of S'. Denote by P the orthogonal projection of C onto S'.

THEOREM 1. *If $T\neq T^*$, then $Tg=T^*g$ if and only if g is in S.*

THEOREM 2. *If V is the restriction of $(I-P)T$ to S, then $V^*=V$.*

INDICATION OF PROOF OF THEOREM 1. From [2] one has that if g is in C, then the member f of C' so that $Lf=g$ is such that

$$\begin{bmatrix} f(t) \\ p(t)f'(t)\end{bmatrix}=\int_a^b\begin{bmatrix}K_{11}(t,j)K_{12}(t,j)\\K_{21}(t,j)K_{22}(t,j)\end{bmatrix}\begin{bmatrix}0\\g\end{bmatrix}$$

for all t in $[a, b]$ ($j(x)=x$ if x is in $[a, b]$) where