UNIFORMLY BOUNDED REPRESENTATIONS OF SL(2, C)

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1. **Introduction.** In [1] Kunze and Stein construct a family of continuous representations of SL(2, R) with the following properties: the representations act on a *fixed* Hilbert space; they are indexed by a complex parameter, and depend analytically on that parameter; included among them are the principal and complementary series representations; and they are uniformly bounded. By applying suitable convexity and Phragmén-Lindelöf type arguments to these bounds and the Plancherel formula for the group, they derive some important applications to harmonic analysis on SL(2, R).

Later, in [2], Kunze and Stein construct a family of representations of SL(n, C) having the same properties as described above (except that they depend analytically on $n-1$ complex variables). However, the uniform bounds obtained are not sufficient to prove any results concerning harmonic analysis on SL(n, C). The author has modified their construction, in the case of $G=\mathrm{SL}(2, C)$, so that it more closely resembles the method used in [1]. As a result, one obtains much sharper estimates on the uniform bounds. One of the consequences is the remarkable fact: Convolution by an $L_p(G)$ function, $1 \leqq p < 2$, is a bounded operator on $L_2(G)$.

2. **Uniformly bounded representations of** SL(2, C). Consider the multipliers

$$\phi(g, z, n, s) = (\beta z + \delta)^{-n} \left| \beta z + \delta \right|^{n-2-2s},$$

$$g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in G = \mathrm{SL}(2, C), \quad z \in C, \quad n \in Z$$

and $s = \sigma + it$ a complex number.

Define the multiplier representations $g \rightarrow T(g, n, s)$, given for f on the complex plane, by

$$T(g, n, s): f(z) \rightarrow \phi(g, z, n, s) f((\alpha z + \gamma)/(\beta z + \delta)).$$

Then the nontrivial irreducible unitary representations of G are:

(a) Principal series: $g \rightarrow T(g, n, it)$, $n \in Z$, $t \in R$, $f \in L_2(C)$;

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