FOURIER SERIES ON THE RING OF INTEGERS IN A p-SERIES FIELD

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1. In this note we will describe a natural setting for harmonic analysis on the dyadic group, 2^ω, (also known as the Walsh-Paley group) and give a few illustrative results. Details and proofs will appear elsewhere.

The dyadic group is viewed classically as the set of all sequences of zeroes and ones with addition (mod 2) defined pointwise, and is supplied with the usual product topology. From our point of view, 2^ω will be the additive subgroup of the ring of formal power series in one variable over $GF(2)$.

The subject of this note is harmonic analysis on the ring of integers, $\mathfrak{O}$, in the field, K (called a p-series field), of formal Laurent series (with finite principal part) in one variable over $GF(p)$, where p is a prime. Such a field K is a particular instance of a local field; that is, a locally compact, totally disconnected, nondiscrete, complete field. The p-adic fields are other examples of local fields. The results in this note have extensions to Fourier series on the ring of integers in any local field and also to multiple Fourier series. These extensions will not be given here.

The idea that 2^ω might be an instance of a ring of integers in a local field developed in a conversation with E. M. Stein.

2. Let the prime p be fixed. An element $x\in K$ is represented as $x=\sum_{-\infty}^{+\infty}a_\nu\mathfrak{p}^\nu$, $a_\nu=0$ for ν small enough, or equivalently as $x=\sum_k^\infty a_\nu\mathfrak{p}^\nu$, $a_\nu=0, 1, \cdots$, or $p-1$ (viewed as elements of $GF(p)$). Addition and multiplication is given by the usual operations in the ring of formal power series over $GF(p)$. A topology is given on K by constructing basic neighborhoods $N_{x,k}=\{y=\sum b_\nu\mathfrak{p}^\nu: b_\nu=a_\nu,\ \nu<k\}$, for each $k\in\boldsymbol{Z}$ and $x=\sum a_\nu\mathfrak{p}^\nu$. With this topology K is a locally compact, totally disconnected, nondiscrete, complete field.

We embed $GF(p)$ in K in the obvious way. The series representations are then convergent series in K and are unique.

The ring of integers $\mathfrak{O}=\{x: x=\sum_{\nu=0}^\infty a_\nu\mathfrak{p}^\nu\}$ is the unique maximal compact subring of K.

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