RESIDUALLY FINITE ONE-RELATOR GROUPS

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Communicated by Michio Suzuki, May 16, 1967

Introduction. It seems to be commonly believed that the presence of elements of finite order in a group with a single defining relation is a complicating rather than a simplifying factor. This note is in support of the opposite point of view, lending respectability to the

CONJECTURE A. *Every group with a single defining relation with nontrivial elements of finite order is residually finite.*

In order to put our results in their proper setting let us define $\langle l, m\rangle$ to be the group generated by a and b subject to the single defining relation $a^{-1}b^l ab^m = 1$:

$$\langle l, m\rangle = (a, b;\, a^{-1}b^l ab^m = 1).$$

Adding a third parameter we define

$$\langle l, m; t\rangle = (a, b;\, (a^{-1}b^l ab^m)^t = 1).$$

Let $\mathcal{L}$ be the class of those groups $\langle l, m\rangle$ satisfying $|l| \neq 1 \neq |m|$, $lm \neq 0$, and l and m relatively prime. Furthermore, let $\mathfrak{M}$ be the class of these groups $\langle l, m; t\rangle$ satisfying the conditions imposed above on l and m, and in addition the extra two conditions $t > 1$, and l, m and t relatively prime in pairs. The point of our initial remark is that $\mathfrak{M}$ looks more complicated than $\mathcal{L}$. Actually $\mathcal{L}$ is quite a nasty class of groups. Indeed the main result of [1] is that every group in $\mathcal{L}$ is isomorphic to one of its proper factor groups, i.e. nonhopfian. Since finitely generated residually finite groups are hopfian (A. I. Mal'cev [2]) no group in $\mathcal{L}$ is residually finite. Our contribution to Conjecture A is that the groups in $\mathfrak{M}$ *are residually finite.*

THEOREM 1. *Every group in the class* $\mathfrak{M}$ *is residually finite.*

In fact even more is true.

THEOREM 2. *If l, m, t are relatively prime in pairs ($l \neq 0 \neq m$) and if t is a power of a prime p ($t \neq 1$) then the group $\langle l, m; t\rangle$ is residually a finite p-group.*

Conjecture A seems difficult. A somewhat easier related conjecture is

[1] Support from the National Science Foundation is gratefully acknowledged. The author is a Sloan Fellow.