TOPOLOGICAL DYNAMICS IN COSET TRANSFORMATION GROUPS

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Communicated by G. A. Hedlund, July 6, 1966

1. **Introduction.** In [1], the actions of the 1-parameter subgroups on coset spaces of certain types of Lie groups were studied with regard to various properties occurring in topological dynamics. A central question was to determine when these actions were minimal and distal. By a considerable use of various structure theorems of Lie groups, it was shown that if the Lie group is connected simply-connected and nilpotent, and the subgroup is discrete and syndetic, then the action of any 1-parameter subgroup is always distal [1, IV, Theorem 3]. Moreover [1, IV, Theorem 5], the action is minimal for those 1-parameter subgroups induced by elements of a comeager subset of the Lie group. In this paper, we use only the properties of topological groups and various results in topological dynamics to generalize the first result. Moreover, we show that the second result can be formulated more generally in the setting of a topological group, and we recover the result in an alternate way.

Let (X, T) be a transformation group. A subset A of T is *(left) syndetic* if there is a compact subset K of T such that $T = AK$. A point x of X is *transitive* if $\text{cl}(xT) = X$. If some point of X is transitive, then (X, T) is a *point-transitive* transformation group. If every point of X is transitive, then (X, T) is a *minimal* transformation group. If X is a uniform space and $x, y \in X$, then x and y are *proximal* provided that for every index α of X there exists $t \in T$ such that $(xt, yt) \in \alpha$. The transformation group (X, T) is *distal* if every pair of distinct points of X is not proximal. As a general reference for these notions, see [2].

Let G be a topological group and let H be a subgroup of G. Then $H\backslash G$ will denote the space of right cosets of H in G. The *coset transformation group of G induced by H* is the transformation group $(H\backslash G, G)$ with action $(Hf, g) \to Hfg$.

Throughout this paper, G will denote a topological group, and H will denote a (left) syndetic closed subgroup of G. We will utilize right-handed functional notation.

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[1] These results form a part of a doctoral dissertation written at Wesleyan University under the supervision of Professor W. H. Gottschalk. During this period, the author held a National Science Foundation Cooperative Graduate Fellowship.