REPRESENTATION THEORY OF CENTRAL TOPOLOGICAL GROUPS

BY SIEGFRIED GROSSER[1] AND MARTIN MOSKOWITZ[2]

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1. **Introduction.** We employ the notation and terminology introduced in the paper "On Central Topological Groups," p. 826 of this Bulletin. As announced there, the representation theory of $[Z]$-groups (as well as their structure theory) generalizes and unifies in a natural fashion that of compact groups on one hand and of locally compact abelian groups on the other.

The following basic definitions will be used throughout the exposition: (1) Let G be a topological group. Consider continuous finite-dimensional irreducible unitary representations ρ of G on the complex Hilbert space V_ρ; denote the degree of ρ by d_ρ and the identity map on V_ρ by I_{d_ρ}. Form equivalence classes of these representations, with respect to unitary equivalence, and choose one representation from each class. We denote by $\mathfrak{R}$ the totality of all such representations. (2) If $\rho\in\mathfrak{R}$ we denote by ρ_{ij} the coordinate functions associated with ρ relative to some orthonormal basis of V_ρ, by χ_ρ the character of ρ, and by $\mathfrak{X}$ the family of all such characters. (3) We denote by $\mathfrak{F}_c$, $\mathfrak{F}_u$, and $\mathfrak{F}_{c_0}$, respectively, the algebras of complex-valued functions on G which are continuous, uniformly continuous,[3] and continuous with compact support; by $\mathfrak{F}_r$ the subalgebra of $\mathfrak{F}_u$ consisting of the representative functions associated with representations in $\mathfrak{R}$, and by $\mathfrak{F}_z$ the subalgebra of $\mathfrak{F}_c$ consisting of the central functions. (4) If $f\in\mathfrak{F}_c$ and $x\in G$ then $x\triangle f$ denotes the conjugate of f by x, i.e., $(x\triangle f)(y)=f(xyx^{-1})$. The restriction of f to a subset S of G is f_S. If S is a subset on which f is bounded, $\|f\|_S$ stands for l.u.b. $\{|f(x)|/x\in S\}$. Finally, $\int_{G/Z}d\dot{x}$ denotes the normalized Haar integral on G/Z and $\int_G dx$ and $\int_Z dz$ are left invariant Haar integrals on G and Z respectively; normalized so that $\int_G=\int_{G/Z}\int_Z$; the associated Haar measures are denoted by $\mu_{G/Z}$, μ_G, and μ_Z. (5) At times, functions on G/Z will be regarded as functions on G.

The next two theorems are technical results required for the investigation.

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[3] Since $[Z]\subseteq[SIN]$, both uniform structures coincide.