LOCALLY FLAT NONEMBEDDABILITY OF CERTAIN PARALLELIZABLE MANIFOLDS

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1. **Introduction and statement of result.** This note is a supplement to the joint papers of R. H. Szczarba and the author [6], [7]. We proved in [6], [7] that for any positive integer $q>1$, there is a differentiable parallelizable manifold M_q of dimension $(2^{4q+1}-8q-2)$ which can be differentiably immersed in Euclidean space of codimension 1 but can not be differentiably embedded in Euclidean space of codimension $8q$. As a consequence, the dimension difference of the best differentiable immersion and the best differentiable embedding in Euclidean space can be arbitrarily large. One may ask the same type of question for topological or combinatorial immersion and embedding. In this note, we shall modify the argument of [6], [7] to show that M_q $(q>1)$ actually has no locally flat topological (hence no combinatorial) embedding in Euclidean space of codimension $8q$. Since we used the normal bundle of a differentiable embedding and Adams' solution of vector field problem [1] in the original proof of [6], [7], differentiability seemed to be essential. However, we shall replace the normal bundle by the normal fibre space of Nash-Fadell-Spivak [9], [4], [10] and use a corollary of Adams' solution of vector field problem that $[\iota_{2^{4q}-1}, \iota_{2^{4q}-1}]$ is not an $(8q+1)$-fold suspension to show the locally flat nonembeddability of M_q $(q>1)$ in Euclidean space of codimension $8q$. The author is indebted to Professor John Milnor for his comments.

Let us first recall the manifolds M_q $(q\geqq 1)$. If ξ and η are sphere bundles with a common base, we use $\hat{\xi}$ to denote the vector bundle associated with ξ and $\xi*\eta$ the sphere bundle associated with $\hat{\xi}\oplus\hat{\eta}$. Let S^{n-1} be the $(n-1)$-sphere where $n=2^{4q}$, $q\geqq 1$. It follows from results of Eckmann [3] and Adams [1] that S^{n-1} has exactly $8q$ independent vector fields. Thus we can find an $(n-8q-1)$-sphere bundle ξ_q over S^{n-1} with a cross section and with the property that $\xi_q * \theta^{8q-1}=\tau(S^{n-1})$, the tangent sphere bundle of S^{n-1}. (Here θ^r denotes the trivial $(r-1)$-sphere bundle.) Let M_q be the total space of ξ_q.

Let M^n and N^m be two topological manifolds. A topological embedding (immersion) $f\colon M^n\to N^m$ is said to be locally flat [4], [5], if

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