ROLLING

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If k is a tame arc in the 3-dimensional half-space $R^3_+=(x, y\ z, t: z\geqq 0, t=0)$ that spans the plane $R^2=(x, y, z, t: z=0, t=0)$ then a locally flat 2-sphere S in the 4-dimensional space $R^4=(x, y, z, t:)$ is generated by k when R^3_+ is rotated about R^2. Nowadays the sphere S is said to be derived from k by *spinning*. By knotting k in various ways, various types of knotted spheres S can be obtained [1], but it is known that not every type of (locally flat) knotted sphere can be so obtained [2].

Some years ago I considered spheres S that are obtained from k by combining the spinning process with a simultaneous rotation of k about its "axis" (in R^3_+). This operation has come to be known as *twist-spinning*. The specific question that I raised at that time—whether the sphere obtained by twist-spinning a trefoil 3_1 (using a simple twist) is actually knotted—has been answered (in the negative) recently by C. Zeeman [3].

In this note I want to introduce another variation of the spinning process, one that I call *roll-spinning*. It is the same as twist-spinning except that instead of twisting, i.e. rotating k about its axis in R^3_+, I *roll* the knot along its axis. This operation (whose name derives from its resemblance to the operation of "rolling a stocking") is somewhat difficult to describe in totally precise terms, and I will content myself here with referring to Figure II, in [3], where it is shown how to roll a figure-eight knot 4_1.

My objective is to show that roll-spinning is not just twist-spinning in disguise (a state of affairs that one might suspect to be so). Specifically I shall show that a *simple roll-spin of* 4_1 *produces a type of knotted sphere* S *that cannot be obtained from* 4_1 *by any twist-spin.*

Figure I gives a projection of 4_1 with the meridian elements of its group G indicated by x, a, b, c. From this figure the presentation

$$(x, a, b, c: ab = bx, cb = ac, cx = xb)$$

is read off in the usual way [2], [4]. If we give 4_1 the twist-spin in which 4_1 is rotated about its axis n times the group Γ_n of the resulting sphere (cf. [2], [3]) has presentation