SOME ELEMENTARY ASPECTS OF MODULAR FUNCTIONS IN SEVERAL VARIABLES

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1. **Introduction.** We shall speak of "elementary" aspects as those which are understood by direct analogy with the rational, classical, "well-known" one-dimensional modular functions, namely, Eisenstein series and theta-functions arising through number-theory. By contrast, some "nonelementary" aspects are those arising in the generalization process in the Siegel theory of modular functions. Actually, some of the problems of current interest are more topological than analytic or number theoretic and in some special instances, functions arising in number theory provide illustrations[2] of topological results (see §16 below), such as compactification parameters.

The basic classical theory [16] can be built around the number of decompositions of an integer into the sum of t-squares (we take $t \equiv 0 \pmod 4$). The number of decomposition depends on functions like $\sigma_{t/2-1}(m)$ the sum of the $(t/2-1)$ powers of the divisors of m by a set of formulas (in §11 below) which are now "classical."

A series of papers of Siegel, Götzky [9], Maass [14], Gundlach [11], etc., has extended this study to the field $Q(\sqrt{5})$. It turns out, however, that a more direct, almost a "word-for-word" analogy can be constructed using $Q(\sqrt{2})$, $Q(\sqrt{3})$ as shown in recent papers [1], [3], [5]. This study is overshadowed, however, by impending difficulties. The fundamental domain involved in $Q(\sqrt{2})$ or $Q(\sqrt{3})$ is actually of a more difficult topological nature than that of $Q(\sqrt{5})$ as shown (see §9 below) by electronic computer results [6], [7]. This difficulty must come to the surface eventually.

At the same time that number theoretic formulas have an incredibly easy generalization by quadratic modular functions, we encounter the difficulty that such functions are not defined on analytic manifolds except in a few artificial cases (as noticed by Gundlach [10]). Thus classical arguments have not traditionally been extended from

An address delivered before the Palo Alto meeting of the Society on April 24, 1965, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors June 7, 1965.

[1] Supported by NSF G-7412. Computer support donated by the Argonne National Laboratories of the USAEC with cooperation of Dr. Wallace J. Givens of the Applied Mathematics Division.

[2] Such results were first announced in tentative form as a short communication to the 1962 International Congress at Stockholm.