INVARIANT FUNCTION ALGEBRAS ON COMPACT SEMISIMPLE LIE GROUPS

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1. Let S^n be the n-sphere, $n>1$. The group G of rotations of n-dimensional Euclidean space acts on S^n. In [1], K. deLeeuw and H. Mirkil studied algebras A of continuous complex valued functions on S^n. Assuming that (i) A contains the constants, (ii) A is uniformly closed, and (iii) A is invariant under G, (i.e. that if $f\in A$ then so does the translate of f by any element of G) they showed that A must be either (i) all continuous functions on S^n, or (ii) just the constants, or (iii) all continuous functions f such that $f(x)=f(x')$ whenever x and x' are antipodal points on S^n. J. Wolf then generalised their result to a wider class of compact connected Riemannian symmetric spaces in [2]. Specifically, Wolf considered the class $\mathcal{C}$ of compact connected Riemannian symmetric spaces X which are *not locally isometric* to a product in which one of the factors is a circle, a group manifold $SU(n)$, $n>2$, $SO(4n+2)$, E_6, or a coset space $SU(n)/SO(n)$, $n>2$, $SU(2n)/Sp(n)$, $n>2$, $SO(4n+2)/SO(2n+1)\times SO(2n+1)$, E_6/F_4, or $E_6/(Sp(4)/\pm I)$.

Here is a short description of Wolf's results. Let X be a compact connected irreducible Riemannian symmetric space. Let G be the component of the identity in the group of isometries $\tilde{G}$ of X. Let Δ be the centralizer of G in $\tilde{G}$. Then Wolf shows that there is a one-one correspondence between uniformly closed G-invariant self-adjoint subalgebras A of $C(X)$ on the one hand, and subgroups Γ of Δ on the other: A being identifiable with the algebra of all f in $C(X)$ such that $f\circ\gamma=f$ for all $\gamma\in\Gamma$. He also showed that if X is in the class $\mathcal{C}$, then every closed G-invariant subspace of $C(X)$ is necessarily self-adjoint. Thus his results give a complete classification of G-invariant closed subalgebras of $C(X)$ containing the constants for X in the class $\mathcal{C}$.

In this note we shall give a short proof of the fact that if X is a compact connected semisimple Lie group and if G is the group $X\times X$ acting on X by two-sided translations, viz. $(u,v): x\to uxv^{-1}$; $u,v,x\in X$, then any G-invariant closed subalgebra $\neq\{0\}$ of $C(X)$ is necessarily self-adjoint and must contain the constants. This extends the results of Wolf to all compact connected symmetric spaces which are group manifolds of semisimple groups. In particular, spaces of this sort in which one of the factors is locally isometric to $SU(n)$, $n>2$, $SO(4n+2)$