SOLUTION OF LINEAR DIFFERENCE AND DIFFERENTIAL EQUATIONS

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1. **Introduction.** The solution of an nth order inhomogeneous difference equation as an explicit function of the starting conditions (without using determinants) has been given by Traub [2], the proof based on a division algebra for sequences. In this note we prove two identities, (1) and (6), which may be used to provide a direct proof of the formulas, (13) and (14), for the solution of inhomogeneous difference or differential equations with constant coefficients. No transforms are required. These solutions are in a form which is felicitous for a number of applications.

2. **Two identities.** Let $P(t)$ be a monic polynomial

$$P(t) = \sum_{j=0}^{n} a_{n-j} t^j, \qquad a_0 = 1,$$

with complex coefficients and with distinct zeros $\rho_1, \rho_2, \cdots, \rho_n$. (The extension to the case of confluent zeros will be treated elsewhere.) Let λ be a nonnegative integer. Then

$$t^\lambda = P(t) \sum_{i=1}^{n} \frac{\rho_i^\lambda}{(t-\rho_i)P'(\rho_i)} + P(t) \sum_{j=0}^{\lambda-1} t^{\lambda-1-j}\Omega(j), \tag{1}$$

where

$$\Omega(j) = \sum_{i=1}^{n} \frac{\rho_i^j}{P'(\rho_i)} \,. \tag{2}$$

We remark that $\Omega(j)$, a symmetric polynomial in the ρ_i, may be obtained by translation from Wronski's aleph function [1],

$$\omega(j) = \sum_{i=1}^{n} \frac{\rho_i^{n-1+j}}{P'(\rho_i)} \,.$$

It is well known that

$$\Omega(j) = \delta_{n-1,j}, \qquad j = 0, 1, \cdots, n-1.$$