BOOK REVIEWS

Lectures on invariant subspaces. By Henry Helson. Academic Press, New York, 1964. 11+130 pp. $5.00

The problem of invariant subspaces is this: does every operator on a non-trivial Hilbert space have a non-trivial invariant subspace? (Explanations: "operator" means bounded linear transformation; "Hilbert space" means complete complex inner-product space; "subspace" means closed linear manifold; "non-trivial", for Hilbert spaces, means of dimension greater than 1; and "non-trivial", for subspaces, means distinct from both $\{0\}$ and the whole space.) The question is thought to be important by some mathematicians and interesting by most; it could be argued that an answer to it (whether yes or no) would be a large step toward a general structure theory for operators on Hilbert spaces. The chief value of the question, however, as of all clearly formulated, unsolved, yes-or-no questions in mathematics, is that of a catalyst and a touchstone. As a catalyst it has precipitated valuable related questions and answers; as a touchstone it has served to measure the extent to which those questions and answers have advanced the theory as a whole.

Helson's book is concerned with the problem of invariant subspaces, some of its special cases, some of its generalizations, and some of the techniques that have yielded partial answers. It is a timely book and will surely be a useful one; it is a highly personal book and a difficult one for all but the specialist; and it is a beautiful book, well conceived and well executed.

There can be little doubt that the subject is currently of interest to many mathematicians. Brodskiĭ [**4**], Brodskiĭ and Livshits [**5**], de Branges [**6**], Kalisch [**13**], Sakhnovich [**18**], and Schwartz [**19**] are actively studying the "subdiagonalization" of operators with "small" imaginary parts. (This list of references is intended to be representative, not exhaustive.) The paper of Foiaş and Sz.-Nagy [**9**] on the existence of non-trivial invariant subspaces for operators A such that neither A^n nor A^{*n} tends strongly to 0 at any non-zero vector has just appeared. Bernstein and Robinson [**2**] (see also [**11**]) have just generalized the Aronszajn-Smith theorem for compact operators [**1**] to operators that are algebraic over the algebra of compact operators. Some of these results have become known too late to be treated by Helson, and none of them appears to be of central interest to him; Schwartz's work is acknowledged with no more than a refer-