DIRECT FACTORS OF (AL)-SPACES[1]

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Let E be a closed sublattice of the (AL)-space L [1, pp. 107–110]. The purpose of this note is to prove that there is a projection of L onto E having norm one. In particular then E is a direct factor of L. To show this we prove auxiliary theorems that the conjugate space E' of E may be "lifted" to L' (see Theorem 2 below) and that there is a projection of $E''=(E')'$ onto $q(E)$, the natural embedding of E in E'', whose norm is one.

The space L' is isometric and lattice isomorphic to a space $C(H)$ of functions continuous on a compact, extremally disconnected Hausdorff space H [4, Theorems 6.3, 6.9, Corollary 6.2]. Then L'' is isometric and lattice isomorphic to the space $R(H)$ of regular measures on H [3, p. 265]. If $x' \in L'$, $x'' \in L''$ correspond to $f \in C(H)$, $\nu \in R(H)$, then $x''(x') = \int_H f d\nu =$ (def. $\nu(f)$). If $\nu \in R(H)$ and $\nu(N)=0$ for each nowhere dense set N then ν is a normal measure. The support A_ν [2, pp. 2, 8, Proposition 3] of such a measure is both open and closed. Let $N(H)$ denote the subspace of normal measures.

THEOREM 1. *The representation of L'' as $R(H)$ maps $q(L)$ onto the space $N(H)$ of normal measures on H. Moreover $\bigcup\{A_\nu \mid \nu \in N(H)\}$ is dense in H (so that H is hyperstonean [2]).*

PROOF. Let $\nu \geqq 0$ correspond to qx for x in L. Let N be a closed nowhere dense set. We prove first that $\nu(N)=0$. Let F be the subset of functions f in $C(H)$ for which $\|f\|=1$, $f \geqq 0$, $f(h)=1$ if $h \in N$. Then F is directed by $\geqq$. This directed set then converges at each such ν to $\inf\{\nu(f) \mid f \in F\}$. Thus F converges on the representation of L in $R(H)$. The directed set of x' in L' corresponding to F then converges pointwise on L to an element y' in L'. If y' corresponds to g in $C(H)$ we have $\nu(g) = \inf\{\nu(f) \mid f \in F\}$ and clearly $g = \inf\{f \in F\}$. Since N is nowhere dense, $g=0$. Thus $\inf\{\nu(f) \mid f \in F\}=0$ so that $\nu(N)=0$. Thus ν is a normal measure.

To prove the second part let A be open and closed in H. For some $\nu>0$ corresponding to qx, $x \in L$, we have $\nu(\chi_A)>0$, where χ_A is the characteristic function of A. Thus A meets the support of ν. Hence H is hyperstonean. The theorem follows immediately from a result of Dixmier [2, p. 21, the corollary and its proof].

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