RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

CONTRACTIBILITY OF CERTAIN SEMIGROUPS

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1. It has long been known that a compact, connected, triangulable topological group, G, must have its Euler-Poincaré number $\nu(G)=0$. The analogous result is not true if G is replaced by S, a topological semigroup with identity. (Witness the unit interval under multiplication.) However, we will show: If S is a compact, connected, triangulable semigroup with identity, then $\nu(S)=0$ or $\nu(S)=1$ and, in the latter case, S is contractible.

2. We follow, in part, the terminology of [1].

Let S be any topological semigroup, then R denotes a minimal right ideal of S, if such exists.

Let us say that a semigroup S satisfies $*$ if $x\in S$ implies there exists a $y\in S$ such that $xy=y$.

We recall that a space is contractible if the identity mapping is homotopic to a constant map.

LEMMA. *Let S be a compact, arcwise connected topological semigroup with identity element e. If S satisfies $*$, then S is contractible.*

PROOF. It is known [1] that S has a minimal right ideal R and that $a\in R$ implies $aR=R=aS$. Fix any $x\in R$, then by $*$ above, there is a $y\in S$ such that $y=xy\in xS=R$. If $z\in R$, then there is a $z'\in R$ such that $yz'=z$; thus $xz=xyz'=yz'=z$. It follows that $xy=y$ for any $x,y\in R$.

Let $i\colon S\rightarrow S$ be the identity mapping and let $x\in R$. Since S is arcwise connected, there is an arc from e to x; let $p\colon I\rightarrow S$ be such an arc (I denotes the unit interval), with $p(0)=e$ and $p(1)=x$.

Define

$$h\colon S\times I\rightarrow S$$

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