SOME THEOREMS ON BOUNDED HOLOMORPHIC FUNCTIONS

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The purpose of this note is the announcement of some results in the theory of bounded holomorphic functions on finite open Riemann surfaces. The proofs are too long to be included; they will be published elsewhere. Special cases of some of the results of this note are to be found in [4].

We consider a fixed finite open Riemann surface R. We thus assume that R is contained as an open set in some compact Riemann surface R_0 and that $R_0\backslash R$ consists of finitely many topological bordered surfaces. We assume further that ∂R, the boundary of R with respect to R_0, consists of the m analytic simple closed curves $\Gamma_1, \cdots, \Gamma_m$.

Denote by $H_\infty[R]$ the algebra of all functions holomorphic and bounded on R. Given the norm $\|\cdot\|_R$ defined by

$$\|f\|_R = \sup\{\,|f(\zeta)| : \zeta \in R\},$$

$H_\infty[R]$ is a Banach algebra. We shall denote by $\mathfrak{M}$ the maximal ideal space of $H_\infty[R]$; $\mathfrak{M}$ will be regarded as the space of all nonzero complex homomorphisms together with the weak* topology. There is a natural embedding of R in $\mathfrak{M}$ given by $\zeta \to \phi_\zeta$ where $\phi_\zeta(f) = f(\zeta)$ for all $f \in H_\infty[R]$ and all $\zeta \in R$. The assumptions on the structure of R guarantee that this correspondence is a homeomorphism.

Our first result is

THEOREM 1. *Let $f_1, \cdots, f_n$ be elements of $H_\infty[R]$ which satisfy $|f_1(\zeta)| + \cdots + |f_n(\zeta)| \geq \delta$ for some $\delta > 0$. Then there exist $g_1, \cdots, g_n \in H_\infty[R]$ such that $f_1(\zeta)g_1(\zeta) + \cdots + f_n(\zeta)g_n(\zeta) = 1$ for all $\zeta \in R$.*

As is known [3, p. 163], this theorem is equivalent to the assertion that R is dense in $\mathfrak{M}$ in the sense that the set of all homomorphisms of form ϕ_ζ is dense in $\mathfrak{M}$. It is this latter fact that our proof establishes. We first obtain the result for the case that R is an annulus. The proof then proceeds along the general lines of the proof of Theorem B of [4]. A different proof of Theorem 1 has been given in [1].

Making use of Theorem 1 in the form of the density of R in $\mathfrak{M}$ and of certain of the constructions in the proof, we are able to obtain some

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