COHOMOLOGY AND DEFORMATIONS OF ALGEBRAIC STRUCTURES

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Communicated by Raoul Bott, December 19, 1963

Gerstenhaber has recently initiated a theory of deformations of associative algebras [4]. The methods and results of Gerstenhaber's work are strikingly similar to those in the theory of deformations of complex analytic structures on compact manifolds. In this note we shall indicate how some of Gerstenhaber's ideas can be reformulated within a framework designed to exploit this similarity, that of graded Lie algebras.

In the work of Kodaira-Nirenberg-Spencer [7] and Kuranishi [8] on the existence of deformations of complex analytic structures, a basic role is played by a certain equation [7, p. 452, equation (3)′] among the 1-dimensional elements of a graded Lie algebra; this equation expresses the integrability conditions for almost complex structures. Our basic observation is that a wide class of algebraic structures on a vector space can be defined by essentially the same equation in an appropriate graded Lie algebra. Among the structures thus obtained are Lie algebras, associative algebras, commutative and associative algebras, extensions of algebras (of any of the above types), and representations of algebras. Our main result, Theorems B and C, is a precise algebraic analogue of a theorem of Kuranishi [8] on deformations of complex analytic structures. Roughly speaking, it states that the set of all structures near a given one can be described in terms of certain cohomology groups, which are defined naturally by means of the graded Lie algebra. In some of the cases mentioned above, these cohomology groups are standard in homological algebra, in others they seem to be new. We also obtain an analogue, Theorem A, of the rigidity theorem of Frölicher and Nijenhuis [2] for deformations of complex analytic structures.

This note contains only an outline of results. A detailed exposition will appear elsewhere.

1. Graded Lie algebras and cohomology. Let K be a field of characteristic $\neq 2$. Let $E=\sum_{n\geqq 0} E^n$ be a graded vector space over K. Let

[1] The authors acknowledge partial support received from an Office of Naval Research contract at the University of Washington. The first named author is a Fulbright Fellow at the University of Amsterdam and also received partial support from NSF GP 1905 at the University of Pennsylvania. The second named author is an NSF Postdoctoral Fellow.