RESEARCH PROBLEMS

1. Daihachiro Sato: *Function theory.*

A. Does there exist a transcendental entire (meromorphic) function which has (1) algebraic values at all algebraic points and has (2) transcendental values at all transcendental points? (The proposer constructed a transcendental entire function with condition (1) alone in [4], using the method similar to that in [1]. The question of the existence of a transcendental entire function with condition (2) alone is open.)

B. Let

$$\begin{aligned}\phi(r) &= \max_n \frac{r^n}{\Gamma(n+1)}\\ &= \exp\left\{r - \frac{1}{2}\log r - \frac{1}{2}\log 2\pi + \frac{1}{24r} + \sum_{k=2}^{m}\frac{C_k}{r^k} + O\left(\frac{1}{r^{m+1}}\right)\right\}.\end{aligned}$$

Find the coefficients C_k explicitly. Are there infinitely many k with $C_k=0$ as in the case of Stirling's formula for $\Gamma(r)$? (The C_k can be calculated successively, but we want to have C_k as a function of k as in the case of Stirling's formula.) $\phi(r)$ gives precise dividing line for the growth $M(r)$ of Hurwitz entire functions (i.e., entire functions $f(z)$ with $f^{(n)}(0)=$ integer, $n=0, 1, 2, \cdots$) below which one finds only polynomials [2; 3; 5].

References

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2. S. Kakeya, *Notes on the maximum modulus of a function*, Tôhoku Math. J. **10** (1916), 68–70.

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4. D. Sato, *A simple example of transcendental entire function which together with all its higher derivatives assumes algebraic values at all algebraic points*, Proc. Amer. Math. Soc. **14** (1963), 996.

5. E. G. Straus and D. Sato, *Rate of growth of Hurwitz entire functions and integer valued entire functions*, Bull. Amer. Math. Soc. **70** (1964), 303–307.

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