A PROBLEM IN PARTITIONS RELATED TO THE STIRLING NUMBERS

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Let

$$S(n,r) = \frac{1}{r!}\sum_{s=0}^{r} (-1)^{r-s}\binom{r}{s}s^n$$

denote the Stirling number of the second kind and put

$$A_n(x) = \sum_{r=0}^{n} S(n, r)x^r.$$

In a recent paper [1] the writer has determined the factorization (mod 2) of the polynomial $A_n(x)$.

Put

$$c_{nr} = S(n+1, r+1);$$

then we have

$$c_{n,2r} \equiv \binom{n-r}{r} \pmod 2 \qquad (0 \leqq 2r < n),$$

$$c_{n,2r+1} \equiv \binom{n-r-1}{r} \pmod 2 \qquad (2r+1 \leqq n).$$

For fixed n, let $\theta_0(n)$ denote the number of odd $c_{n,2r}$ and $\theta_1(n)$ the number of even $c_{n,2r}$. Then

$$\theta_0(2n+1) = \theta_0(n), \qquad \theta_0(2n) = \theta_0(n) + \theta_0(n-1)$$

and

$$\theta_1(n+1) = \theta_0(n).$$

Moreover we have the generating function

$$\sum_{n=0}^{\infty} \theta_0(n)x^n = \prod_{n=0}^{\infty} (1 + x^{2^n} + x^{2^{n+1}}).$$

It follows that $\theta_0(n)$ can also be defined as the number of partitions

$$n = n_0 + n_1\cdot 2 + n_2\cdot 2^2 + \cdots \qquad (0 \leqq n_j \leqq 2)$$

subject to the conditions