INVERSIVE PLANES OF EVEN ORDER

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Communicated by A. M. Gleason, June 10, 1962

1. **Results.** An *inversive plane* is an incidence structure of *points* and *circles* satisfying the following axioms:

I. *Three distinct points are connected by exactly one circle.*

II. *If P, Q are two points and c a circle through P but not Q, then there is exactly one circle c' through P and Q such that $c \cap c' = \{P\}$.*

III. *There are at least two circles. Every circle has at least three points.*

For any point P of the inversive plane $\mathfrak{J}$, the points $\neq P$ and the circles through P form an affine plane $\mathfrak{A}(P)$. If $\mathfrak{J}$ is finite, all these affine planes have the same order (number of points per line); this integer is also termed the order of $\mathfrak{J}$. An inversive plane of order n consists of n^2+1 points and $n(n^2+1)$ circles; every circle contains $n+1$ points, and any two points are connected by $n+1$ circles.

Let $\mathfrak{P}$ be a projective space of dimension $d>1$ (we shall only be concerned with $d=2$, 3, and we do not assume the theorem of Desargues if $d=2$). A point set $\mathfrak{C}$ in $\mathfrak{P}$ is called an *ovoid* if

I'. *Any straight line of $\mathfrak{P}$ meets $\mathfrak{C}$ in at most two points;*

II'. *For any $P \in \mathfrak{C}$, the union of all lines x with $x \cap \mathfrak{C} = \{P\}$ is a hyperplane.*

(This is called the tangent hyperplane to $\mathfrak{C}$ in P.) It is straightforward to prove that the points and the nontrivial plane sections of an ovoid in a three-dimensional projective space form an inversive plane. The purpose of the present note is the announcement, and an outline of proof, of the following partial converse:

THEOREM 1. *Every inversive plane of even order n is isomorphic to the system of points and plane sections of an ovoid in a three-dimensional projective space over* GF(n).

We list three immediate corollaries: If $\mathfrak{J}$ is an inversive plane of even order n, then (i) n is a power of 2, (ii) for any $P \in \mathfrak{J}$, the affine plane $\mathfrak{A}(P)$ is desarguesian, and (iii) $\mathfrak{J}$ satisfies the bundle theorem ("Büschelsatz," cf., e.g., [2]).

The proof of Theorem 1, to be outlined in §2 below, shows also that every automorphism (incidence preserving permutation) of an inversive plane of even order can be extended to a collineation, leaving the representing ovoid invariant, of the appropriate projective space. Together with recent results of Tits [9], [10], this leads to a complete