ON THE CONTINUITY OF INVERSE OPERATORS IN (LF)-SPACES[1,2]

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Though the well-known equivalence of continuity of the inverse operator and permanent solvability of the adjoint one can be verified for a very wide class of linear topological spaces, it often turns out to be not a very deep fact and one can hardly expect to get much help using the equivalence to solve problems out of the classical analysis. This paper gives another characterization of the permanent solvability of the adjoint operator, which seems to be less trivial and involving more of the special internal structure of (LF)-spaces. Proofs of the results presented here will appear in the Studia Mathematicae.

Consider an (LF)-space (X, τ) [1]. A subspace of X which is metrizable and complete provided with the relativization of topology τ is called an (F)-*subspace* of (X, τ); (X, τ) is called an (LB)-*space* iff every (F)-subspace of (X, τ) is a Banach space. We say that an (LF)-space (Z, ρ) *majorizes* an (LF)-space (X, τ) iff Z is algebraically a subspace of X and to every (F)-subspace U of (Z, ρ) there corresponds an (F)-subspace Y of (X, τ) such that $U \subset Y$ and the natural injection of (U, ρ) into (Y, τ) is continuous (we denote the relativizations of topologies by the same letters).

A *projective component* (ξ) of an (LF)-space (X, τ) is an (LB)-space (X_ξ, ξ) such that (X, τ) majorizes (ξ) and every (F)-subspace of (ξ) is contained in a ξ-closure of an (F)-subspace of (X, τ); the projective component (ξ) is *reflexive* iff every (F)-subspace of (ξ) is a reflexive Banach space. A family Φ of projective components of (X, τ) form a *basis* of projective components of (X, τ) iff to every τ-continuous pseudonorm $|\cdot|$ on X there correspond $(\xi) \in \Phi$ such that for every (F)-subspace U of (X, τ) the identical injection of (U, τ) into $(U, |\cdot|)$ is continuous and to every (ξ_1), $(\xi_2) \in \Phi$ there correspond $(\xi_3) \in \Phi$ such that (ξ_3) majorizes both (ξ_1) and (ξ_2).

Let (X, τ) and (X_1, τ_1) be two (LF)-spaces with bases of projective components Φ and Φ_1 respectively and let A be a continuous mapping of (X_1, ξ_1) into (X, ξ).

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