LOCALLY FLAT, LOCALLY TAME, AND TAME EMBEDDINGS

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Communicated by Deane Montgomery, May 23, 1963

1. **Introduction.** Brown [1] has shown that an S^{n-1} embedded in a locally flat manner in S^n is flat and hence tame in S^n. Bing [2] and Moise [3] have shown that locally tame subsets of 3-manifolds are tame. However, in the general case, it is not known whether a manifold N embedded in a locally flat manner in a triangulated manifold M or a polyhedron P embedded in a locally tame manner in a triangulated manifold M are tame in M. Partial solutions to both of these problems have been obtained by the author and will be stated in §3 of this paper. I have been informed by R. H. Bing that Herman Gluck has obtained similar results.

2. **Definitions and notations.** Let N^k be a combinatorial k-manifold. Then $(N^k)^r$ will denote the rth barycentric subdivision of N^k. If α is a k-simplex of $(N^k)^r$ and α'' is the union of all simplexes of $(N^k)^{r+2}$ contained in α, then C_α will denote the closed simplicial neighborhood of $|\alpha''|$, the polyhedron of α'', in $(N^k)^{r+2}$. That is C_α is the union of all closed simplexes in $(N^k)^{r+2}$ that meet $|\alpha''|$. Since α'' is collapsible, C_α is a combinatorial k-ball [4].

The statement that f is a locally flat embedding of a k-manifold N^k in an n-manifold N^n, means that each point of $f(N^k)$ has a neighborhood U in N^n such that the pair $(U, U\cap f(N^k))$ is homeomorphic to the pair (R^n, R^k).

Two definitions of locally tame will now be given.

DEFINITION 1. Let N be a manifold topologically embedded in a triangulated manifold M. N is locally tame if for each point p of N, there exists a neighborhood U of p in M and a homeomorphism h of $\overline{U}$ into M, such that $h[\mathrm{Cl}(U\cap N)]$ is a polyhedron in M.

DEFINITION 2. Let P be a polyhedron topologically embedded in a triangulated manifold M. P is locally tame if for each point p of P, there exists a neighborhood U of p in M and a homeomorphism h of $\overline{U}$ into M, such that $h|\mathrm{Cl}(U\cap P)$ is piecewise linear with respect to a fixed triangulation T of P.

Let K be a complex topologically embedded by f in a triangulated n-manifold N^n and let $\epsilon>0$. Suppose there exists an ϵ-homeomorphism h of N^n onto itself such that if $U_\epsilon(f(K))$ denotes the set of points in N^n whose distance from $f(K)$ is less than ϵ, then