A CHARACTERIZATION OF COMMUTATIVE GROUP ALGEBRAS AND MEASURE ALGEBRAS

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1. **Introduction.** The group algebra, $L(G)$, of a locally compact Abelian group, G, is the Banach algebra of all complex-valued functions on G integrable with respect to Haar measure on G, with convolution as multiplication. The measure algebra, $M(G)$, of G is the Banach algebra of all finite complex regular Borel measures on G, with convolution as multiplication.

In this note we give a characterization of those commutative Banach algebras which are the group algebras of locally compact Abelian groups, and a similar characterization of the measure algebras. Sketches of proofs will be given. Complete details will be published elsewhere.

The results reported in this note are contained in a dissertation submitted in partial fulfillment of the requirements for the Ph.D. at Columbia University. We are happy to have this opportunity to express our gratitude to Professor Richard V. Kadison for his encouragement of this work and for many helpful comments and suggestions.

2. **Preliminaries.** Define an abstract complex L-space, B, to be a partially ordered complex Banach space such that the real subspace, R, of B generated by the positive elements of B is a real Banach lattice under the order of B restricted to R, and such that the following hold:

(I) If $x, y \in R$ and $x, y \geqq 0$, then $\|x+y\| = \|x\| + \|y\|$.

(II) If $x, y \in R$ and $x \wedge y = 0$, then $\|x+y\| = \|x-y\|$.

(III) If $x \in B$, then there exist unique $\mathrm{Re}(x)$, $\mathrm{Im}(x)$ in R such that $x = \mathrm{Re}(x) + i\,\mathrm{Im}(x)$.

(IV) If for x in B we define $|x| = \bigvee\{\mathrm{Re}(e^{i\theta}x) : \theta \in [0, 2\pi]\}$ then $\|x\| = \|\,|x|\,\|$.

Then in analogy with Kakutani's theorem [5] on the concrete representation of abstract real L-spaces, B is isometrically isomorphic to complex $L^1(X, m)$ for some measure space (X, m).

Let B be any Banach space, and let f be any nonzero linear functional on B. Let $P(f) = \{x : f(x) = \|f\|\,\|x\|\}$. Then $P(f)$ is a cone in B and so defines a partial order on B in the usual way.

Now let A be any complex commutative Banach algebra, with