ON DIFFERENTIABLE IMBEDDINGS OF SIMPLY-CONNECTED MANIFOLDS

BY J. LEVINE

Communicated by W. S. Massey, June 12, 1963

1. **Introduction.** We will be concerned with the problem of imbedding (differentiably) a closed *simply-connected* n-manifold M in the m-sphere S^m. According to [3], this problem depends only upon the homotopy type of M, in a "stable" range of dimensions. We obtain an explicit equivalent homotopy problem.

We also consider the problem of determining whether two imbeddings of M in S^m are isotopic (see [3] for basic definitions). A "homotopy condition" for deciding this question will also be obtained, again in a "stable" range of dimensions.

All manifolds, imbeddings and isotopies are to be differentiable. If M, V are manifolds with boundary and f is an imbedding of M in V, it will always be understood that $f(M)\cap\partial V=f(\partial M)$ and the intersection is transverse.

2. **Imbedding theorem.** M will, hereafter, denote a closed simply-connected n-manifold, $n>4$. Suppose f imbeds M in S^m; then we can define the normal plane bundle ν_f and, by a construction of Thom [10], an element $\alpha_f\in\pi_m(T(\nu_f))$, where $T(\nu_f)$ is the *Thom space* (see [10]) of ν_f. We call the pair (ν_f, α_f) the *normal invariants* of f. The existence of an imbedding, in particular, implies the existence of an $(m-n)$-plane bundle ξ whose Thom space is *reducible* in the sense of [1]. It follows from [1] that this property of M is a homotopy invariant and such a bundle ξ must be, a priori, stably fiber homotopy equivalent to the stable normal bundle of M.

Let M_0 denote the complement of an open disk in M.

THEOREM 1. *Suppose $2m\geqq 3(n+1)$ and ξ is an $(m-n)$-plane bundle over M stably equivalent to the stable normal bundle of M, such that $T(\xi)$ is reducible. Then there is an imbedding f of M in S^m such that ν_f is fiber homotopy equivalent to ξ:*

(a) *Over M if $n=6$, 14 or $n\not\equiv 2$* mod 4.

(b) *Over M_0 if $n\equiv 2$* mod 4.

It is to be expected that the conclusion of (a) is valid for all n. The difficulty in the proof arises from the lack of a satisfactory general definition of the *Arf invariant* (see [7]). In certain special cases, e.g., if M is a π-manifold or $\pi_i(M)=0$ for $2i<n$, we can obtain the conclusion of (a).