INFINITE MEASURE PRESERVING TRANSFORMATIONS WITH "MIXING"

BY S. KAKUTANI AND W. PARRY[1]

Communicated by W. S. Massey, June 24, 1963

1. **Introduction.** It is well known that a transformation T which preserves a finite measure has the mixing property

$$(1.1)\qquad T^{(k)} = T\times T\times \cdots \times T \ (k \text{ times}, k\geqq 2) \text{ is ergodic}$$
$$\text{if and only if } T \text{ is weakly mixing [1].}$$

The purpose of this note is to give, for each positive integer k, an example of a transformation T which preserves a σ-finite infinite measure with the property,

$$(1.2)\qquad T^{(k)} \text{ is ergodic but } T^{(k+1)} \text{ is not ergodic.}$$

We also give an example of a transformation T which preserves a σ-finite infinite measure with the property

$$(1.3)\qquad T^{(k)} \text{ is ergodic for each } k = 1, 2, \cdots.$$

A transformation T with property (1.2) is said to have ergodic index k and a transformation T with property (1.3) is said to have infinite ergodic index. For completeness, we say that a nonergodic transformation has zero ergodic index.

Thus, for each $k=0, 1, 2, \cdots, \infty$, infinite measure preserving transformations exist with ergodic index k, unlike finite measure preserving transformations which assume ergodic indices 0, 1, ∞ only.

The examples are taken from Gillis [2], and are Markov transformations derived from "centrally biased random-walks."

2. **Markov transformations preserving a σ-finite infinite measure.** Let

$$P = \|p(i,j)\|, \qquad i,j = 0, \pm 1, \pm 2, \cdots$$

be a stochastic matrix with only one ergodic class, i.e.,

$$p(i,j) \geqq 0, \qquad \sum_{j=-\infty}^{\infty} p(i,j) = 1,$$

and for each (i, j) there exists $n>0$ for which $p^n(i, j)>0$ where $P^n=\|p^n(i, j)\|$. Assume also that there exists a left eigenvector

[1] Research supported in part by NSF G-25222.