NORMAL CONGRUENCE SUBGROUPS OF THE $t \times t$ MODULAR GROUP[1]

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Let Γ denote the group of rational integral $t \times t$ matrices of determinant 1. If n is a positive integer, $\Gamma(n)$ denotes the *principal congruence subgroup* of Γ of *level n*, consisting of all elements of Γ congruent modulo n to a scalar matrix. The subgroup of $\Gamma(n)$ consisting of all elements of Γ congruent modulo n to the identity matrix is denoted by $\Gamma_1(n)$. Then $\Gamma(n)$, $\Gamma_1(n)$ are normal subgroups of Γ. A subgroup G of Γ containing a principal congruence subgroup $\Gamma(n)$ is termed a *congruence subgroup*, and is said to be of *level n* if n is the least such integer. Notice that $\Gamma_1(n)$ is not in general a congruence subgroup, according to the definition above.

Let p be a prime. Let $\mathrm{SL}(t, p)$ denote the group of $t \times t$ matrices with elements from $\mathrm{GF}(p)$ and determinant 1, and let $H(t, p)$ denote the normal subgroup of $\mathrm{SL}(t, p)$ consisting of all scalar matrices. Then

$$\mathrm{SL}(t, p) \cong \Gamma/\Gamma_1(p), \qquad H(t, p) \cong \Gamma(p)/\Gamma_1(p),$$

and $\mathrm{SL}(t, p)$, $H(t, p)$ are of orders

$$p^{t^2-1} \prod_{j=2}^{t} (1 - p^{-j}), \quad (t, p - 1)$$

respectively. In his book on the linear groups [1] Dickson proves that for $t>2$, $H(t, p)$ is a maximal normal subgroup of $\mathrm{SL}(t, p)$ and this of course implies that $\Gamma(p)$ *is a maximal normal subgroup of* Γ. This result is used to prove the theorem that follows:

THEOREM 1. *Suppose that $t>2$. Then every normal congruence subgroup of odd level of Γ is a principal congruence subgroup.*

The theorem is also true for $t=2$, if the level is prime to 6. (The case $t=2$ for the group of linear fractional transformations has been treated in [3].) Since the structure of the proof of Theorem 1 is identical with that of the proof for $t=2$ given in [3], we only indicate the points of difference, and refer the reader to [3] for full details. The proof is arranged for an induction and what is actually proved is the slightly more general theorem that follows:

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