References

1. Jacques Herbrand, *Recherches sur la théorie de la démonstration*, Travaux de la Société des Sciences et des Lettres de Varsovie, Classe III sciences mathématiques et physiques, no. 33 (1930).

2. ———, *Sur le problème fondamental de la logique mathématique*, Comptes Rendus des séances de la Société des Sciences et des Lettres de Varsovie, Classe III sciences mathématiques et physiques, no. 24 (1931).

3. John van Heijenoort, Editor, *Jacques Herbrand, Écrits Logiques*, Presses Universitaires, Paris (to appear)

HARVARD UNIVERSITY AND
PRINCETON UNIVERSITY

SIMPLY INVARIANT SUBSPACES[1]

BY T. P. SRINIVASAN

Communicated by Edwin Hewitt, March 18, 1963

Let L^1, L^2 denote respectively the spaces of summable and square summable functions on the circle group and H^1, H^2 their subspaces consisting of those functions whose Fourier coefficients vanish for negative indices. A closed subspace M of L^1 or L^2 is "invariant" if

$$\chi M \subset M$$

and "simply invariant" if the above inclusion is strict, where χ is the character

$$\chi(x) = e^{ix}.$$

The structure of simply invariant subspaces is known, namely, they are precisely the subspaces of the form qH^1 or qH^2 (respectively) where q is a measurable function of modulus 1 a.e. Beurling [1] first proved this for subspaces $M \subset H^2$; for $M \subset H^1$, this is due to de Leeuw-Rudin [5]; for $M \subset L^2$, due to Helson-Lowdenslager [3] and for $M \subset L^1$, due to Forelli [2]. In [3] Helson-Lowdenslager also gave a simple proof of the H^2 case, free of function theoretic considerations. Using their arguments Hoffman [4] extended this result to *simply invariant* subspaces of $H^2(dm)$ defined over *logmodular algebras*. In this paper we prove this result for simply invariant subspaces of $L^2(dm)$ and $L^1(dm)$ over logmodular algebras; the results of the previous authors follow as a corollary. The proofs of the previous authors

---

[1] This work was done while I held a visiting appointment at the University of California, Berkeley.

I thank Professors Helson and Ju-kwei Wang for the useful discussions I had with them.