THE GEOMETRY OF IMMERSIONS. I

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We hereby announce some general methods for higher order differential geometry. Our main tools are a generalization of the general position theory of Whitney and Thom, and a characteristic class theory for higher order bundles having given higher order connections.

The second part of this announcement will deal with applications of this machinery to some problems of geometric singularities. One application will be to count the number of umbilic points on an immersed hypersurface. Full details will appear in a separate publication.

1. **pth order osculating maps.** It is known [4] that on each smooth manifold X a sequence of smooth vector bundles $\{T_k(X)\}_{k=1,2,\cdots}$ over X can be canonically constructed. $T_p(X)$ is called the *bundle of pth order tangent vectors over* X, and $T_1(X)$ is just the tangent bundle of X. These bundles furthermore satisfy short exact sequences

$$0 \to T_{p-1}(X) \to T_p(X) \to 0^p T_1(X) \to 0$$

where $0^p T_1(X)$ denotes the p-fold symmetric tensor product of the tangent bundle. It is also known that to each smooth map f between manifolds X and Y there is canonically defined a *pth order differential* $T_p(f): T_p(X) \to T_p(Y)$ which is a homomorphism of smooth vector bundles covering f. For each smooth f there is the following family of commutative diagrams of vector bundles with exact rows,

$$\begin{array}{ccccccccc} 0 & \longrightarrow & T_{p-1}(X) & \longrightarrow & T_p(X) & \longrightarrow & 0^p T_1(X) & \longrightarrow & 0 \\ & & \downarrow T_{p-1}(f) & & \downarrow T_p(f) & & \downarrow 0^p T_1(f) & & \\ 0 & \longrightarrow & T_{p-1}(Y) & \longrightarrow & T_p(Y) & \longrightarrow & 0^p T_1(Y) & \longrightarrow & 0. \end{array}$$

If the dimension of X is n, then the fiber dimension of $T_p(X)$ is

$$\nu(n, p) = n + \binom{n+1}{2} + \cdots + \binom{n+p-1}{p}.$$

The smooth sections $S(T_p(X))$ of $T_p(X)$ will be called the *pth order*

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