RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

TWO THEOREMS ON NONLINEAR FUNCTIONAL EQUATIONS IN HILBERT SPACE

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Let H be a Hilbert space, with real or complex scalars. A function $F: H \rightarrow H$ is called *monotonic* provided, for any $x_1, x_2 \in H$, we have $\operatorname{Re}\langle x_1 - x_2, Fx_1 - Fx_2 \rangle \geqq 0$. If ($\geqq$) is replaced by ($>$), it is *strictly* monotonic, and if 0 is replaced by $c\|x_1 - x_2\|^2$, with $c > 0$, it is *strongly* monotonic. Examples are: the gradient of a convex (resp. strictly or strongly convex) function, the negative of a linear dissipative operator, a linear operator satisfying $\operatorname{Re}\langle x, Fx \rangle \geqq c\|x\|^2$ (the hypothesis of a form of the Lax-Milgram Lemma), and so on.

A variant, due to F. E. Browder, of a theorem of the author [5, Corollary to Theorem 4] asserts that a continuous, everywhere-defined strongly monotonic function has a continuous everywhere-defined inverse. (Browder has also generalized the theorem.) These results are used in the proofs of the following theorems:

THEOREM 1. *If F is everywhere-defined, continuous, and monotonic, and satisfies for some real M*

$$\|x\| > M \text{ implies } \operatorname{Re}\langle x, Fx \rangle \geqq 0 \tag{1}$$

then the equation $Fx = \theta$ has a solution, which is unique if F is strictly monotonic.

THEOREM 2. *If K and F are everywhere-defined, continuous, and monotonic, K is linear, and in addition F is a bounded operator and satisfies* (1), *then the "Hammerstein equation" $x + KFx = \theta$ has a solution; the solution is unique if either K or F is strictly monotonic.*

A (nonlinear) operator is called "bounded" if it maps bounded sets into bounded sets.

A VARIANT ON THEOREM 2. *If K is strongly monotonic, the hypotheses of boundedness of F can be dropped from Theorem 2.*

The proofs will appear in [2]. The application of Theorem 2 to non-