INTEGRALS DEVISED FOR SPECIAL PURPOSES

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About forty-five years ago Professor T. H. Hildebrandt wrote a paper (Hildebrandt [1]) on integrals related to and extensions of the Lebesgue integral. At that time the Lebesgue integral, first invented in the preceding decade, had already displaced the Riemann integral from the monarchial position that it had occupied for over a generation. However, the subject was still new enough so that it was possible for Professor Hildebrandt to give an intelligible account of each one of the generalizations or extensions of the Lebesgue integral then extant, and to finish the job within a reasonable space of time. To do this today for the whole field of integration, covering all topics adequately, and still remaining within the bounds of a one hour talk, would be an impossible task. I have therefore chosen to confine myself to one particular line of integration theory. I shall pass by all developments of new types of integral for the sake of increasing the generality of the process of integration. Likewise, I shall pass by all research designed to increase our knowledge of already existent forms of integrals. The research that I intend to report on is extremely diverse in nature, but all the developments that I shall mention have one aspect in common. In each case a mathematician needed an integration process to attain some goal, found that none of the traditional types of integration did exactly what he wanted and proceeded to invent an integration method capable of producing the results that he wished.

A common feature of the integrals about which I wish to speak is that they are all what might be called "second growth" integrals. Probably none of them would have been thought of at all if the man who defined it had not been thoroughly familiar with an assortment of standard integration procedures, and a number of them in fact cannot even be defined without using concepts that themselves would never have arisen if it had not been for the development of the theory of the Lebesgue integral. This means that the one-time dominance of the Riemann integral has not been replaced by a similar dominance of any other integral. Competence in this field demands familiarity with a large assortment of concepts connected with procedures that might be called integration.

A compact example of the sort of theory that I am speaking of

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