# ISOTROPIC INFINITELY DIVISIBLE PROCESSES ON COMPACT SYMMETRIC SPACES

BY RAMESH GANGOLLI

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Let $G$ be a connected compact Lie group, $K$ a compact subgroup, such that $G/K$ is a Riemannian symmetric homogeneous space. (See [1] for terminology and notation.) We fix once and for all a $G$-invariant metric on $G/K$. $\boldsymbol{D}(G/K)$ will stand for the algebra of those differential operators on $C^\infty(G/K)$ which are invariant under the action of $G$. $\boldsymbol{S}(K\backslash G/K)$ stands for the semi-group, under convolution as product, of probability measures on $G$ which are bi-invariant under the action of $K$. It is known [2] that $\boldsymbol{S}$ is a commutative semi-group. Let $\phi_n(x)$, $n=0, 1, 2, \cdots$ be the $K$-spherical functions on $G$ [1, pp. 398 ff.]. For $\mu\in\boldsymbol{S}$, we define the Fourier coefficients $\hat{\mu}(n) = \int_G \phi_n(x)d\mu(x)$. A measure $\mu\in\boldsymbol{S}$ is said to be infinitely divisible if for each positive integer $k$, $\mu=\nu^k$ for some $\nu\in\boldsymbol{S}$. The purpose of this note is to set down characterizations of such measures. Our main theorem is

THEOREM 1. *Let $\mu\in\boldsymbol{S}$ be such that $\hat{\mu}(n)\neq 0$ for any $n$. Then $\mu$ is infinitely divisible if and only if*

$$\hat{\mu}(n) = \exp - \left(\lambda_n + \int_{G-\{e\}} (1-\phi_n(x))M(dx)\right)$$

*where $M(dx)$ is a non-negative measure on $G$, bi-invariant under $K$ and such that $\int_G r^2 M(dx) < \infty$, $r$ being the distance of $x\in G$ from $e$; and $\lambda_n$ is the eigenvalue corresponding to the eigenfunction $\phi_n$ of an elliptic, second order operator $D$ in $\boldsymbol{S}(G/K)$. Further, $D$ and $M$ are uniquely determined by $\mu$.*

Let $\mu\in\boldsymbol{S}$ be such that $\hat{\mu}(n)\neq 0$ for any $n$. Call $\mu$ a generalized limit for each $n$, $\hat{\mu}(n)=\lim_{j\to\infty}\prod_{r=1}^{k_j}\hat{\mu}_{jr}(n)$ with $\mu_{jr}\in\boldsymbol{S}$, $|\hat{\mu}_{jr}(n)-1|\to 0$ as $j\to\infty$ uniformly for $1\leqq r\leqq k_j$.

THEOREM 2. *The class of infinitely divisible measures of Theorem* 1 *coincides with the class of generalized limits.*

As a corollary to Theorem 1, we can state results about the Fourier coefficients $\hat{\mu}_t(n)$ of one-parameter subsemigroups of $\boldsymbol{S}(K\backslash G/K)$ or what is the same thing, describe $K$-isotropic stochastic processes on