A DUALITY THEORY FOR CONVEX PROGRAMS WITH CONVEX CONSTRAINTS

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The existence of a solution to the problem of minimizing a convex function subject to restriction of the variables to a closed convex set in n-space ("convex programming") has been characterized (for suitable differentiability conditions) by the Kuhn-Tucker theorem [5]. In general, no dual programming problem (not involving the variables of the direct problem) has been associated with this situation except in the linear programming case, and very recently by E. Eisenberg in [3], for homogeneity of order one in the function and linear inequality constraints, and by R. J. Duffin [2] in an inverse manner for a highly specialized problem.

Starting with a little known paper of A. Haar [4] in the light of current linear programming constructs (e.g., "regularization" [1]), we effect a generalization of these ideas (with maximal finite algebra and minimal topology) so that a dual theory practically as straightforward as linear programming theory is obtained, and which includes a dual theorem covering the most general convex programming situation (e.g. no differentiability conditions qualifying the convex function or constraints, or homogeneity, etc.).

This general theorem is made possible by associating a suitably restricted, usually infinite-dimensional space problem with the minimization problem in n-space instead of the usual association of another finite m-space problem. The space we use is a "generalized finite sequence space" (g.f.s.s.), defined with respect to an index set I of arbitrary cardinality as the vector space, S, of all vectors $\lambda = [\lambda_i: i \in I]$ over an ordered field F with only finitely many nonzero entries.

Such spaces possess the following key characteristics for linear programming of ordinary n-spaces. Let V be a vector space over F and consider a collection of vectors: P_0, $P_i: i \in I$ in V. Let R be the subspace spanned by these vectors, and let

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